Relativistic Particle Kinematics

The universe is filled with the cosmic background radiation, photons with an average energy of about 10^{-3} eV. Energetic processes can also create high energy photons – gamma rays. Although the cross section is small, interactions do take place, causing electron-positron creation for example. Since this destroys gamma ray photons above a certain threshold energy, one expects an observable feature in the gamma ray energy spectrum. To find the threshold it is convenient to use the four-momenta, allowing you to solve simultaneously the conditions of energy and momentum conservation.

a) Show that for a massive particle the square of the four-momentum $p^\alpha p_\alpha = -m^2$.

b) Find the square of the total four-momentum of the left hand side of the reaction $\gamma\gamma' \rightarrow e^+e^-$, i.e. $(p^\alpha + p'^\alpha)(p_\alpha + p'_\alpha)$ in terms of the energies E and E'.

c) Since the total four-momentum in the reaction must be conserved, the result in b) gives the total amount of mass equivalent available, $-m^2$, for massive particle production. In order to create an electron and proton, each with mass 511 keV, what is the minimum gamma ray energy necessary?