SENSITIVITY TO ANALYSIS TECHNIQUE IN COSMOLOGICAL RESULTS FROM TYPE IA SUPERNOVAE

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Draft version February 21, 2003

ABSTRACT

We compare the three methods currently used to analyze the light curves of Type Ia supernovae, and compare the results from those methods for low and high redshift supernovae. We have fit the data from Riess et al. (1998) using the method in Perlmutter et al. (1997) and Perlmutter et al. (1999) in order to have three sets of results for the same data. Statistical tests find no evidence for differences between high and low redshift samples. The different analysis methods agree well overall. The main systematic differences between the methods, which are mainly due to the data before maximum, are present for both high and low redshift supernovae. The cosmological results reported in earlier papers are therefore unaffected by the differences in analysis methods.

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1. INTRODUCTION

For many years Type Ia supernovae have been thought to be good standard candles. Kowal (1968) produced the first Hubble diagram for Type I supernovae, finding a dispersion of 0.6 mag for the absolute magnitudes. Hubble diagrams for the subclass Ia were published by Tammann & Leibundgut (1990), van den Bergh & Pazder (1992), and Sandage & Tammann (1993) with dispersions between 0.2 and 0.5 mag. However there were early indications that Type Ia supernovae are not identical. Pskovskii (1977) found that different Type I supernovae had different rates of decline, and Branch (1981) suggested that the rate of decline of Type Ia supernovae was correlated with expansion velocity and peak absolute magnitude. Phillips (1993) provided the first concrete evidence of this, finding the absolute magnitudes of nine well-observed supernovae to be correlated with the rate of decline.

Using supernovae from the Calan-Tololo survey (Hamuy et al., 1996d), Hamuy et al. (1996a) confirmed the correlation between light curve shape and absolute magnitude, and produced a family of light curves (Hamuy et al., 1996c) labelled by their decline rate parameter. These template curves were used by Hamuy et al. (1996b) to obtain a Hubble diagram for supernovae in the Hubble flow. Riess, Press, & Kirshner (1995, 1996) developed an alternative method to obtain templates continuously parametrized by the deviation of the absolute magnitude from that of a standard supernova, using all bands simultaneously.

The Supernovae Cosmology Project (SCP), obtained estimates of the cosmological parameters Ω_m and Ω_Λ using the Calan-Tololo supernovae and 7 high redshift ($z > 0.3$) supernovae observed by the SCP (Perlmutter et al., 1997). Perlmutter et al. (1999) analyzed a larger set of 42 high redshift SCP supernovae and the Calan-Tololo sample. These papers used an analysis method which linearly stretched or compressed the timescale of a single light curve template by an amount s called the stretch factor. The High-Z Team (Riess et al., 1998) used two different analysis methods (the method used in Hamuy et al. (1996b) and the MLCS method developed in Riess, Press, & Kirshner (1996)) on 10 high redshift and 10 low redshift supernovae from their own search (Schmidt et al., 1998) and the Calan-Tololo sample. Both Riess et al. (1998) and Perlmutter et al. (1999) came to the conclusion that the Type Ia supernova data requires a non-zero cosmological constant.

There have been attempts to investigate possible systematic effects which might influence the results for cosmology. The systematic dimming of supernovae at high redshifts compared to what would be expected in a zero cosmological constant universe could occur if high redshift supernovae were extinguished more than low redshift supernovae (Aguirre, 1999) or if there was some form of evolution in supernovae properties that was highly correlated with redshift (Drell, Loredo, & Wasserman, 2000). In particular Drell et al. suggest evidence of evolution in the data set used by Riess et al. (1998) and Perlmutter et al. (1999).

We have performed the same analysis as used in Perlmutter et al. (1997, 1999) on the data used in Riess et al. (1998), so that we have results for the same set of data from the three different analyses (the Hamuy et al. (1996b) method, the stretch-fitting method and the MLCS method). This paper is a first look at the differences between the methods as used in previous analyses.

In the next Section we describe the three analysis methods used to fit light curve data and introduce the parameters used to describe the supernovae. In Section 3 we specify the dataset used and some details of how we fit the Riess et al. (1998) data with the Perlmutter et al. (1997) method.

Section 4 looks at the differences in results from the different analysis methods for the low redshift data, which are well-observed with small errorbars and many points on each light curve. We explore the systematic differences between the results from different methods, and discuss in Sections 5.1 and 5.2 how these differences arise from differences in the methods, and how they are affected by the quality of the data.

In Section 6 we use both parametric and non-parametric methods to test whether or not the high and low redshift populations have the same properties. We compare both the actual light curve fit parameters, and their differences between methods. We find in contradiction to Drell, Loredo, & Wasserman (2000) that there is no evidence against the hypothesis that the two populations have the same distributions of these quantities. We conclude with a discussion of implications in Section 7.

2. LIGHT CURVE FITTING METHODS

The magnitude-redshift relationship for any object is

$$m - M = 5\log(d_L(z)) + 25 \tag{1}$$

where m is the apparent bolometric magnitude for the object, M the absolute bolometric magnitude, $d_L \equiv \mathcal{D}_L/H_0$ the luminosity distance and H_0 the Hubble constant. It can equivalently be written

$$m - \mathcal{M} = 5\log(\mathcal{D}_L(z)) \tag{2}$$

where $\mathcal{M} \equiv M - 5\log(H_0) + 25$, to explicitly eliminate the H_0 dependence from the equation. Note that $\mathcal{M}$ is observable from m and z of supernovae in the Hubble flow without knowing H_0 or absolute magnitude. The cosmological parameters other than H_0 appear in $\mathcal{D}_L$:

$$\mathcal{D}_L(z) = \frac{c(1+z)}{\sqrt{|\Omega_k|}} S\left(\sqrt{|\Omega_k|}\int_0^z (\Omega_m(1+z')^3 + \Omega_{DE}(1+z')^{3(1+w_{DE})} + \Omega_k(1+z')^2)^{-1/2}dz'\right) \tag{3}$$

where $S(x)$ is $\sin(x), x, \sinh(x)$ for closed, critical and open cosmologies respectively, Ω_m is the present day fraction of the critical energy in matter, Ω_{DE} is the fraction of critical energy in dark energy or a cosmological constant, w_{DE} defines the equation of state for the dark energy field or is -1 for a cosmological constant and $\Omega_k \equiv 1 - \Omega_m - \Omega_{DE}$ is the fraction of critical energy in curvature. Observations of m and z are used to obtain an estimate for Ω_m, using equations 2 and 3.

For Type Ia supernovae the absolute magnitude has been found to be correlated with the width of the light curve (Phillips, 1993; Hamuy et al., 1995, 1996b). The different analysis methods we look at here employ different ways of implementing this correlation.

All three methods of fitting light curves obtain four parameters: the time of maximum luminosity t_{max} (in the B band, by convention), a measurement of the width of the light curve (or equivalent parameterization), and measurements of the apparent magnitudes in two different bands (rest-frame B and V) which together give an estimate for the distance modulus and extinction. Broadly speaking the distance modulus comes from the average of the magnitudes in the different bands and the extinction comes from the difference.

The observations in this work were made in various filter bands and corrected to the B or V band using cross-filter K-corrections (Kim et al., 1996; Nugent et al., 2002). From now on m_B and m_V represent the K-corrected magnitudes.

The Multicolor Light Curve Shapes (MLCS) method used in Riess, Press, & Kirshner (1995, 1996) and Riess et al. (1998) uses a standard light curve, adding or subtracting correction curves for fainter or brighter supernovae. The parameter equivalent to light curve width is Δ, which is the difference between the absolute magnitude of a "standard" supernova and the absolute magnitude of the supernova in question. This method fits for the distance modulus and extinction directly, rather than first obtaining the apparent magnitudes. In each band X it fits the template $M_X(t)$ plus various amounts of two correction templates $R_X(t)$ and $Q_X(t)$:

$$m_X^{data}(t) = M_X(t - t_{max}) + R_X(t - t_{max})\Delta + Q_X(t - t_{max})\Delta^2 + \mu + A_X \tag{4}$$

where A_X is the extinction in band X. The templates are found by minimizing a χ^2 between them and the data for supernovae with "known" Δ, t_{max}, μ and A_X. This estimation also gives the uncertainties on the templates.

The stretch-factor method used in Perlmutter et al. (1997, 1999) (here called "Stretch") uses a standard template $\mathcal{T}_X$ close to the Leibundgut (Leibundgut et al., 1991) one, and stretches it to give narrower/faster or wider/slower light curves. The fit to the data is

$$f_X^{data}(t) = 10^{-0.4(m_X^{fit} + \mathcal{T}_X((t - t_{max})/s))} + g_{resid(X)} \tag{5}$$

where f_X is the flux data in band X and $g_{resid(X)}$ is a constant fit to account for residual host galaxy light.[1] The template is zero at t_{max}, so that m_X^{fit} is the fit apparent magnitude of the supernovae. Larger stretch factor s means the light curve is slower than the Leibundgut light curve. The standard template for each band is found using a large number of well-observed light-curves (see e.g. Goldhaber et al., 2001). The correlation between magnitude and light curve timescale is implemented via the equation

$$m_B^{fit} + \alpha(s-1) = \mathcal{M}_B + 5\log(\mathcal{D}_L(z)) \tag{6}$$

where α and $\mathcal{M}_B$ are constants, fit for simultaneously with the cosmological parameters in $\mathcal{D}_L(z)$. The dependence on the Hubble constant and absolute magnitude of a standard supernova (M_B^{stand}) are contained in $\mathcal{M}_B$, which is a constant term in this equation. The fit m_B^{fit} against redshift therefore determines the cosmological parameters $\Omega_k, \Omega_m, \Omega_{DE}$ and w_{DE} independently of the Hubble constant and M_B^{stand}. If a correction for host extinction is used it is found from the difference between m_B^{fit} and m_V^{fit} using the standard extinction laws (Clayton & Cardelli, 1988).

The "Template fitting method" (here referred to as "Dm15") used in Hamuy et al. (1995, 1996b) and Riess et al. (1998) uses the parameter Δm_{15}, the difference in B magnitude between rest-frame day 0 (t_{max}) and day 15 to parameterize the timescale of the light curve (Phillips, 1993). It uses several templates labelled by Δm_{15} (Hamuy et al., 1996c). The χ^2 between data and templates for these discrete values of Δm_{15} are interpolated to find the value of Δm_{15} to give a theoretical minimum χ^2 (Hamuy et al., 1995). Note that this χ^2 value does not correspond to an actual template fitting to the data, so the value found for Δm_{15} may not correspond to the actual 15 day decline. Host extinction is found using standard extinction laws (Clayton & Cardelli, 1988).

In this paper we will use the symbol Δ_{corr} for the correction in magnitude needed to bring a supernova to the standard absolute magnitude ($M_B = M_B^{stand} - \Delta_{corr}$). In order to compare results for the different methods, we will use the distance modulus μ, the host extinction A_B in band B and the magnitude correction Δ_{corr}. Extinction in our galaxy is already included in μ, since this is determined separately using dust maps (Burstein & Heiles, 1982; Schlegel et al., 1998). For the MLCS method the magnitude correction is $-\Delta$, for the Stretch method it is $\alpha(s-1)$ and for the Dm15 method it is $-0.784(\Delta m_{15} - 1.1)$ (Hamuy et al., 1996b). As a notational convenience we convert the Stretch fit magnitudes m_B^{fit} to distance moduli using

$$\mu = m_B^{fit} - (M_B^{stand} - \Delta_{corr}) - A_B. \tag{7}$$

where M_B^{stand} is an arbitrary choice of absolute magnitude to put this on a distance scale. Note that the value of M_B^{stand} has no effect on the cosmological parameters of interest ($\Omega_k, \Omega_m, \Omega_{DE}$ and w_{DE}).

Here we are not explicitly comparing the time of maximum light found by the different methods, however it is possible, especially when using sparse data, that there may be a correlation between the time of maximum and other fit parameters. Therefore the sampling of the light curve is an important aspect of the data to look at when considering differences in the results (see Section 5.2).

3. LIGHT CURVE ANALYSIS

In this paper we use a dataset of 80 supernovae analyzed with the Stretch method which can all be put on the same Hubble diagram. This consists of 18 low redshift supernovae from the Calan-Tololo survey (Hamuy et al., 1996d), 10 low redshift supernovae from the CfA Survey (Riess et al., 1999), 10 high redshift supernovae from the High-Z Team (Riess et al., 1998; Schmidt et al., 1998; Garnavich et al., 1998) and the 42 high redshift SCP supernovae (Perlmutter et al., 1999). The High-Z Team, CfA and Calan-Tololo supernovae were used in Riess et al. (1998) to estimate the cosmological parameters, and the SCP and Calan-Tololo data were used in Perlmutter et al. (1999) for the same purpose. For the direct supernova by supernova comparison of the different methods in Section 4 we use the Calan-Tololo, CfA and High-Z Team data only.

[1]Light from the host galaxy has already been subtracted for all three methods. The parameter g_{resid} is an additional check on this subtraction.

K-corrections for the high redshift data are the same for all three methods. For the low redshift data the same K-corrections have been used for the MLCS and Dm15 methods. The results from the low redshift Stretch analysis use K-corrections from spectral information compiled by the SCP, so as to be the same as the results published in Perlmutter et al. (1999).

We have not used the parameter g_{resid} for the High-Z Team data as we do not have the late time points which would identify this parameter. Currently g_{resid} is not important, as the ratio of residual flux to actual flux is always less than 10^{-3}. This translates to a 10^{-3} error in magnitude, much less than other fitting errors. In the future g_{resid} may be important for other datasets, since it is negatively correlated with the magnitude correction due to light curve width.

In this paper we implement the Stretch method as it was used in Perlmutter et al. (1999) for the main comparison of methods so as to be able to comment on the differences between the results of Perlmutter et al. (1999) and Riess et al. (1998). In the primary fit reported in Perlmutter et al. (1999) ("Fit C") supernovae thought to be reddened were excluded as the majority are not expected to be extinguished (Hatano et al., 1998). Other fits in Perlmutter et al. (1999) did include host extinction corrections. Here we also find estimates for host extinction using a prior probability that the extinction must be positive, as in Riess, Press, & Kirshner (1996). The main fit in Perlmutter et al. (1999) used information from the V band only to obtain colors of supernovae, not apparent magnitudes or stretch. The reason for this is that the high redshift data is generally much higher quality in B than in V, therefore the B band determines stretch which can then be used with the V band to obtain the color. In the current analysis we also compare the Stretch results using both B and V curves to estimate distance moduli.

4. DIFFERENCES AT LOW REDSHIFT

Figure 1 shows the distance moduli found for the low redshift supernovae by the three pairs of methods (X,Y), along with the line $\mu^X = \mu^Y$, with X,Y representing MLCS, Dm15 and Stretch. Here we use the Burstein and Heiles (Burstein & Heiles, 1982) estimate of extinction for our galaxy, and no host extinction on the Stretch results, as in Fit C of Perlmutter et al. (1999). Although more recent estimates of the extinction in our own galaxy (Schlegel et al., 1998) were used in the Stretch analysis in Perlmutter et al. (1999), we use the Burstein and Heiles estimates here in order to compare with the MLCS and Dm15 results. Supernovae not used in Fit C of Perlmutter et al. (1999) are excluded from comparisons involving Stretch here. No supernovae are excluded from the MLCS – Dm15 comparison. We see that broadly speaking all methods agree well. To examine them in more detail, from now on we will concentrate on the differences in the distance moduli $\mu^X - \mu^Y$.

In the following sub-sections we describe the differences between the distance moduli and magnitude corrections found by the different methods (sections 4.1 and 4.2). In sections 5.1 and 5.2 we further examine which differences are caused purely by the different methods (and therefore would be present however good the data), which are most sensitive to certain properties of supernovae such as light curve width, and which extra differences are caused due to uncertainty in the data, or a lack of data.

We mention the overall level of agreement between the methods, as measured by the standard deviation of the differences in distance modulus $\sigma(\mu^X - \mu^Y)$, only to compare relatively how the three pairs of methods agree. We do not compare these overall levels to uncertainties on the data, or calculate χ^2 as Drell, Loredo, & Wasserman (2000) do, since these calculations only tell us how well the current data distinguishes between the analysis methods. Since the data sets are actually the same – not just drawn from the same parent distribution – there is no other significance of a high χ^2 or high ratio of $\sigma(\mu^X - \mu^Y)$ to data uncertainty.

4.1. *Differences in Magnitude Correction*

In this section we compare the different values of magnitude correction Δ_{corr} found by the different analysis methods. The magnitude correction is effectively the absolute magnitude for the supernova ($M_B \equiv M_B^{stand} - \Delta_{corr}$). Since the choice of standard supernova is arbitrary, the magnitude corrections are only defined up to an additive constant. Here we shift them to make the average agree between methods (equivalent to redefining the standard supernova). From now on, when we talk about standard templates we mean those corresponding to $\Delta_{corr} = 0$.

Figure 2 shows the magnitude corrections Δ^X_{corr} versus Δ^Y_{corr} for each pair of methods, and Figure 3 shows $\Delta^Y_{corr} - \Delta^X_{corr}$ versus the individual magnitude corrections. We measure the approximate overall level of agreement between two sets of magnitude corrections by the rms scatter in $\Delta^Y_{corr} - \Delta^X_{corr}$. This is 0.17 mag for MLCS versus Stretch, 0.18 mag for MLCS versus Dm15, and 0.08 mag for Dm15 versus Stretch. The Dm15 and Stretch results clearly agree better with each other than with MLCS. We can see this in Figure 3, where the y-axes are the same for all the graphs.

However there are still clear systematic differences between all three methods. The differences in magnitude correction between Dm15 and Stretch are highly correlated with the Dm15 magnitude corrections (see Figure 3, middle left panel). We can account for this by the different formulae used to calculate the magnitude corrections, as will be shown in Section 5.1.

The MLCS magnitude corrections show a double-valued behavior with respect to the other methods, most clearly shown in Figure 2. Half the supernovae in the graph of MLCS versus Stretch lie on a line with slope approximately 1.1, and half lie on a line with slope 2.0.[2] We call these Groups I and II respectively, when referring to them in later sections. In Sections 5.1 and 5.2 we consider possible reasons for this behavior.

4.2. *Differences in Distance Moduli*

One might think that only the Δ_{corr} differ significantly between methods, but in fact the fitted apparent magnitude does change slightly, so we must look at the distance modulus separately. Since all three methods use similar light curves as standard, we should expect their results to agree the most for supernovae with light curves close to the standard. A reasonable hypothesis is that the differences in the distances obtained by the different analysis methods might increase as the shape of the light curve of the supernova diverges from the "standard" template.

Figure 4 shows the difference between the distance moduli for each pair of methods X,Y plotted against the magnitude corrections Δ^X_{corr} and Δ^Y_{corr}. The differences $\mu^X - \mu^Y$ have been centred around zero, since the methods have used different distance scales. All plots have the same y-axis for comparison. One can immediately see that the overall level of agreement is approximately the same between all three pairs of methods. The rms scatter in differences between distance moduli is 0.22 mag between MLCS and Stretch, 0.18 mag between MLCS and Dm15, and 0.20 mag between Dm15 and Stretch.

However for this sample the distance moduli differences between MLCS and Stretch and those between MLCS and Dm15 are correlated with magnitude correction (see upper left and lower right panels in Figure 4), while no such trend is seen in the Dm15 and Stretch comparison. In general, the more extreme the size of the magnitude correction needed, the more the MLCS method disagrees with the other two. This appears to suggest that for light curves which are particularly broad or narrow the distances found for the supernovae by the different methods disagree more, because the template curves used to fit the supernovae become more different as they become broader or narrower.

Since we have already seen a natural split of the supernovae into two groups in terms of the MLCS and Stretch magnitude corrections, we will consider what effect this has on the distances. The graphs in Figure 4 show the supernovae in Group II as filled squares. There is a clear linear relation between the difference in distance modulus and the magnitude corrections for the Group II supernovae:[3]

$$\mu^{MLCS} - \mu^{Stretch} \sim 1.2\Delta^{MLCS}_{corr} \tag{8}$$

Note that if the differences in distance moduli were purely due to differences in magnitude correction we would expect a relation

$$\mu^{MLCS} - \mu^{Stretch} \sim 0.5\Delta^{MLCS}_{corr} \tag{9}$$

[2]We put 11 supernovae in each group for now, and consider later how results change if we change the group membership for the supernovae which are ambiguous.

[3]There is one supernova in Group II, SN1992ag, which does not show this behavior. The reason for this is that it has a large host extinction (0.77 mag found by MLCS and 0.70 found by Stretch) so naturally the difference between the distance moduli is large when the MLCS value is corrected for extinction and the Stretch is not. We look at the results using the extinction-corrected distances in Section 5.1.

since $\mu^{MLCS} - \mu^{Stretch} \sim \Delta_{corr}^{MLCS} - \Delta_{corr}^{Stretch}$ by equation 7 and $\Delta_{corr}^{MLCS} \sim 2\Delta_{corr}^{Stretch}$ for Group II. We are seeing extra differences in distance modulus for the Group II supernovae.

The supernovae in Group I are much more scattered on these graphs. There is no clear relation between distance modulus difference and magnitude correction for these supernovae.

5. REASONS FOR DIFFERENCES

5.1. *Analysis Methods*

In this section we try to account for some of the differences in results directly in terms of differences in analysis methods. To understand the small differences between the Dm15 and Stretch results, we look at (1) the sets of templates used by these methods and (2) the different width-brightness relations used. To check the effect of possible variations of the Stretch method we look at (3) the results obtained for the Stretch method including a positive host extinction and (4) the results obtained for this method using both B and V bands.

Dm15 and Stretch Template Curves:

The template light curves used for the Stretch and Dm15 methods are very similar. It is not possible to show a theoretical fitted light curve for each supernovae that would be found by the Dm15 method, as this fitting procedure interpolates between χ^2 values for standard templates to find Δm_{15} and does not construct a template curve with the minimum χ^2. We can however compare the standard Dm15 templates to those used in Stretch. The five broadest Dm15 curves agree with the corresponding (*i.e.* those with the same Δm_{15}) Stretch templates to within 0.15 mag up to 40 days after maximum light in the rest-frame (over a typical range of 3 mag). This time range covers the vast majority of data points used. All supernovae considered here have light curve widths within the range of these five templates except SN1992br, which is considered an outlier in the Stretch analysis and is not included in the sample we are comparing here. Therefore we would not expect great differences in fit parameters to arise due to the different sets of templates being used in these two methods.

Dm15 and Stretch Width-Brightness Relations:

The width-brightness relation used in the Dm15 method is linear in Δm_{15}. Since the parameters stretch (s) used in Stretch and Δ used in MLCS are presumably not linearly related to Δm_{15}, the width-brightness relations used in these methods will not be linear in Δm_{15}. Therefore we expect to get non-linear relationships between the magnitude corrections from the different methods. This suggests a possible reason for the correlation of the difference in magnitude corrections with individual magnitude corrections, which would be present even if the template curves used to fit the supernovae were the same.

We can calculate the relation between stretch and Δm_{15} for the Stretch templates. This enables us to calculate what magnitude corrections would have been obtained using Stretch templates and the original form of the Phillips relation rather than the stretch-correction parameterization. Figure 5 shows various possible magnitude corrections as a function of Δm_{15}. The straight line is the linear Dm15 magnitude correction $-0.784(\Delta m_{15} - 1.1)$ as used in Hamuy et al. (1996b). This line corresponds to agreement between the magnitude corrections, but we do not expect this since the width-brightness relations used are different. The curve is the Stretch magnitude correction $\alpha(s-1)$ using the non-linear relationship between Δm_{15} and stretch

$$\Delta m_{15} \approx 2.0(s^{-1} - 1) + 1.1, \tag{10}$$

from Perlmutter et al. (1997). This is how we would expect the data to behave if Δm_{15} found by Dm15 and Stretch were the same for each supernova.

The data is also shown, plotted as $\Delta_{corr}^{Stretch}$ (found by Stretch) versus Δm_{15} (found by Dm15). The data agrees better (as measured by sum of squared residuals) with the curve than with the straight line, indicating that it is behaving as expected. We have added a curve showing the expected non-linear dependence of $\Delta_{corr}^{Dm15} - \Delta_{corr}^{Stretch}$ with Δ_{corr}^{Dm15} and $\Delta_{corr}^{Stretch}$ to Figures 2 and 3. Here we can see clearly that the differences between Dm15 and Stretch magnitude corrections for the faintest supernovae can be accounted for by the different width-brightness relations used for the two methods.

It is interesting to note that Hamuy et al. (1995) suggest that the relationship between absolute magnitude and Δm_{15} may become non-linear for the faintest supernovae. Compare our Figure 5 with their Figure 4, where they have plotted $-M_B^{abs}$ against Δm_{15}. The sense of the non-linear relation used in the Stretch method is what is needed to give higher luminosities for the faintest supernovae. The data set of this paper cannot strongly distinguish between the linear and non-linear width-brightness relations however.

So it seems that the variation of the difference in magnitude corrections for the Dm15 and Stretch methods with the individual magnitude corrections as shown in Figure 3 can be explained by the different formulae used for the magnitude corrections for the two methods.

Stretch with Host Extinction:

So far in this paper we have not considered host extinction corrections on the Stretch distances. This is so that we can compare to one of the primary analyses (Fit C) in Perlmutter et al. (1999), where supernovae thought to have host extinction were excluded from the analysis, as most supernovae we see are not expected to have extinction (Hatano et al., 1998). Here we look at the results including host extinction corrections, as in Fit E of Perlmutter et al. (1999). After extinction is applied the slope of the stretch-magnitude relation becomes $\alpha = 1.7$ (because magnitude correction and host extinction are correlated).

Note that the increase in α increases the range of magnitude corrections for the Stretch method. In Drell, Loredo, & Wasserman (2000) it is pointed out that the Stretch method obtains a considerably smaller range in magnitude correction than either of the other methods. However when the correct value of α is used for host extinction corrected results, the range of magnitude corrections found by all three methods are similar.

The double-valued nature of the MLCS and Stretch magnitude corrections does not change of course, since the stretch s does not. The overall level of agreement between the MLCS and Stretch methods remains approximately the same: the spread in distance moduli differences is 0.19 mag. The differences are still correlated with the magnitude corrections. The slope of the best fit relation is reduced ($\mu^{MLCS} - \mu^{Stretch} \sim 0.8\Delta_{corr}^{MLCS}$), since the differences caused directly by the differences in magnitude corrections are smaller, but the correlation caused by the difference in template curves remains. Correcting for host extinction cannot compensate for this.

The overall level of agreement between the Dm15 and Stretch distance moduli remains the same when host extinction is included; there is still no significant correlation with magnitude correction. The conclusions we made above about the differences between the Dm15 and Stretch magnitude corrections coming from the different width-brightness relations used do not change with α, since there we were essentially testing the agreement between Δm_{15} as found by the two methods. The Dm15 formula for magnitude corrections ($-0.784(\Delta m_{15} - 1.1)$) is equivalent to $\alpha = 1.6$, using linear approximations. So the slope of the width-brightness relation found by the Stretch method including host extinction agrees very well with that used in the Dm15 method. This is interesting, since the Dm15 width-brightness relation was found using only low redshift supernovae and the Stretch relation uses both high and low redshift.

Stretch using both B and V Bands:

The other big difference between the methods is that so far we've applied the Stretch estimates of distance modulus using apparent magnitude from the rest-frame B band only, while for the other methods we used both B and V bands. This was done in Perlmutter et al. (1999) because the high redshift SCP data were more frequently sampled in B than in V.

The double-valued behaviour of the magnitude corrections remains when both bands are used in the Stretch method, as the stretch parameter changes very little.

When distance moduli are calculated using both B and V bands, we find very similar results to those using only the B band. Figure 6 shows histograms of the differences in distance moduli between the

Stretch method and the other two, for Stretch results using only B (Stretch (B)) and both B and V (Stretch (BV)). Also shown is a histogram of the differences between the Stretch (B) and Stretch (BV) distance moduli. Clearly the distance moduli found by the Stretch method using only the B band are much closer to those found by the Stretch method using both B and V than either of the MLCS or Dm15 methods using B and V. Therefore the fact that only B curves were used in the Stretch analysis cannot be causing the differences in results.

5.2. *Properties of Individual Supernovae*

In Section 5.1 we showed that the decline rate parameter Δm_{15} is a predictor of the difference in magnitude corrections between the Dm15 and Stretch methods, simply because of the different width-brightness relations used. The differences in distance modulus for these two methods are not correlated with magnitude correction however. Therefore we will look for some other property of the supernovae measurements (rather than the methods themselves) which predicts differences in distance moduli.

We also saw in Section 4.2 how the differences in distance moduli between MLCS and Stretch and between MLCS and Dm15 depend on the magnitude corrections, partly due to differences in the magnitude corrections themselves, and probably partly due to the template curves being different even when the magnitude corrections are the same. However we have not seen the reason for the disagreements in magnitude corrections. In this section we will also look for properties of the supernovae which predict the "double-valued" behavior of the magnitude corrections found by these methods.

Below we look at a list of properties of the data to see if they correlate with these differences. We look at (1) host galaxy morphology, (2) goodness of fit to the Stretch templates, (3) size of the errorbars on the light curve data, (4) whether or not there are observations before maximum light, (5) whether or not there are observations after 40 days after maximum light and (6) number of points on the light curve.

Host Galaxy Morphology:

Host galaxy morphology tends to be correlated with luminosity (Hamuy et al., 1995), the fainter supernovae being found in ellipticals and the brighter ones in spirals. Hence we would not expect to be able to predict the extra differences in magnitude correction and distance modulus using host galaxy morphology. We have checked this however, as the relation between morphology and intrinsic luminosity has a fairly large scatter. We find that both Groups I and II of supernovae mentioned in Section 4.1 extend over the whole range of host galaxy types. We also find no dependence of the difference in Dm15 and Stretch distance moduli on host galaxy type, as we would expect.

Light Curve Goodness of Fit:

One might expect the supernovae for which the minimum χ^2 for the light curve fit is bad to have greater differences between the methods than those which all the methods fit well. If the data is such that none of the templates fit it well, there is no reason to expect the methods to give the same results. In fact this is not the case. There is no significant correlation between any of the differences in distance modulus or differences in magnitude correction and the χ^2 for the Stretch fit.

Size of Errorbars:

Another possibility related to the goodness of the light curve fit is to look at the dependence of differences on the size of the errors on the light curve data. If there are some parts of the template curves which are in some way "incompatible" with each other this could force differences in results for supernovae with good data. In particular we might expect that supernovae with smaller errorbars might be in one or other of the two groups with respect to the MLCS and Stretch magnitude corrections. This does not seem to be the case. There are supernovae with large and small errorbars in both groups. There is no significant correlation between difference in Dm15 and Stretch distance modulus and size of errorbars, so this cannot be causing the extra spread in difference in distance modulus beyond that of the magnitude corrections for this pair of methods.

Observations before Maximum Light:

An important aspect of the data to look at is the sampling of the light curve. When we divide the supernovae into two groups defined by the two lines in the MLCS - Stretch magnitude correction plane, we immediately see that all but one of the supernovae which have observations before maximum light are in Group II (see Figure 7). This is also shown in Figures 2 and 3, where the filled circles are the supernovae with observations before maximum and open circles are those without. There is only one supernovae in Group I with observations before maximum light.[4] Apparently we can say that if a supernova has points before maximum it is very likely to have a particular relationship between MLCS Δ and Stretch stretch:

$$-\Delta \sim 2(s-1). \tag{11}$$

It appears that light curves *without* points before maximum may be fit with the "wrong" magnitude correction due to insufficient sampling.

We have refit the light curve data with the Stretch method ignoring all points before maximum to see if the Stretch method is sensitive to the existence of points before maximum. There is no change in the double-valued behaviour of the MLCS and Stretch magnitude corrections. The one supernova in Group I that has points before maximum changes its magnitude correction by 0.10 mag. All the supernovae in Group II have at most an insignificant change of 0.02 mag in $\Delta_{corr}^{Stretch}$, which we can see by looking at Figure 2 cannot bring them into line with the rest of the sample. It appears that it is the MLCS templates that are sensitive to whether or not a supernova has observations before maximum light.

Having seen this, it is interesting to go back and look at the Dm15 and Stretch comparison, to see how supernovae without observations before maximum light behave. Looking at the second panel in Figure 2, we see that all the supernovae with observations before maximum are on the positive side of the expected non-linear relationship between the Dm15 and Stretch magnitude corrections. This may be another example of double-valued behaviour (albeit much less obvious). Again the 0.02 mag sensitivity in Stretch to whther or not points before maximum are included cannot change this behaviour.

Late Time Observations:

For the MLCS method all points on the curve after 40 days in the restframe were discarded, but they were used in the Stretch analysis. However, supernovae with points which were discarded for the MLCS analysis divide equally into those where the magnitude corrections agree and those where they don't, so this does not seem to be an issue for the disagreements between these methods.

Number of Data Points in Light Curve:

We also test whether the number of data points on the light curve has an effect. Supernovae with points before maximum light tend to have more points overall, so we use the number of observations after maximum so that we do not test the same property twice. The average number of observations after maximum is 14.7 for Group I and 16.5 for Group II. We calculate by boostrap resampling the distribution of average number of points after maximum for 11 supernovae (the number of supernovae in each group) and find it to be approximately Normal with mean 17.2 and standard deviation 2.1. Therefore the number of points after maximum for Groups I and II show no significant differences. We conclude that number of observations after maximum does not affect the relation between magnitude corrections for MLCS and the other two methods.

6. HIGH REDSHIFT DATA

Having looked at the differences in the results of the different analyses for the high quality low-redshift data, we now turn to the high-redshift data. We are interested in whether there could be some evolution of supernovae properties leading to a redshift dependence of the relation between absolute magnitude and light curve width or other quantity.

[4] For the supernovae with the most negative magnitude corrections the choice of Group I or II is ambiguous. However for the data where the 2 slopes are clearly different, the existence or otherwise of observations before maximum distinguish between them (see Figure 2).

It is important to remember that a different distribution of light curve width or absolute magnitude at high redshift from that at low redshift would not intrinsically be a problem for estimating cosmological parameters. The question is whether the width of the light curve predicts the absolute magnitude in the same way at high redshift as at low redshift. However it is still interesting to see if high and low redshift supernovae have different properties. Since the different fitting methods must be sensitive to slightly different aspects of supernova physics, a change in properties from high to low redshift might show up as a change in the differences between the methods at high and low redshift. We therefore consider both the original light curve fit parameters from each method and the difference in fit parameters as found by different methods.

In Drell, Loredo, & Wasserman (2000) it is suggested that the mean of the difference between MLCS and Dm15 distance moduli may be different at high and low redshift. We find $\mu^{MLCS} - \mu^{Dm15} = -0.07 \pm 0.04$ mag for the low redshift supernovae and -0.09 ± 0.09 mag for the high redshift. The two methods here may not be on the same distance scale, so $\mu^{MLCS} - \mu^{Dm15}$ does not have to be zero, but the values for low and high redshift are clearly consistent, providing no evidence for a difference between low and high redshift.

The average difference in distance moduli are consistent between the low and high redshift samples for all three pairs of methods. We have performed Kolmogorov-Smirnov tests, which look at the maximum difference between the cumulative distributions at low and high redshift, on the individual magnitude corrections Δ^X_{corr}, the differences in magnitude corrections between the methods $\Delta^X_{corr} - \Delta^Y_{corr}$ and the differences in distance moduli $\mu^X - \mu^Y$ for all the analysis methods X and Y. None of these quantities show a significant difference in distribution for low and high redshift (the lowest p-value is 0.21).

Figures 8, 9 and 10 show the same quantities as Figures 2, 3 and 4 with the high redshift data added to the low redshift sample. The differences $\mu^X - \mu^Y$ and $\Delta^X_{corr} - \Delta^Y_{corr}$ have been shifted to make their means zero, in the same way as for the low redshift data. Another claim that Drell, Loredo, & Wasserman (2000) make is that the difference in distance moduli between the MLCS and Dm15 results appears to be correlated with absolute magnitude averaged from the MLCS and Dm15 fits at high redshift but not at low redshift. Looking at absolute magnitude is the same as looking at the magnitude correction dependence, due to the width-brightness relation ($M_B \equiv M_B^{stand} - \Delta_{corr}$). To test the claim, we have carried out a two-dimensional Kolmogorov-Smirnov type test (Peacock, 1983; Fasano & Franceschini, 1987) on the distribution of $\{\mu^{MLCS} - \mu^{Dm15}, (\Delta^{MLCS}_{corr} + \Delta^{Dm15}_{corr})/2\}$, comparing this distribution at low and high redshift. (This is the non-parametric equivalent of comparing correlation coefficients.) There is no significant difference.

We have also performed two-dimensional Kolmogorov-Smirnov type tests on the low and high redshift distributions of the other combinations of differences in distance modulus and magnitude correction versus individual magnitude correction for all three methods. None of these distributions are significantly different for low and high redshift either (lowest p-value is 0.55).

Another quantity to consider is reddening. We have looked at the distributions of color excess $E(B-V)$ at high and low redshift. Kolmogorov-Smirnov tests find no significant differences between high and low redshift (lowest p-value 0.08). For the comparison between high and low redshift samples for the Stretch method, we can use all the data we have. We have done the same Kolmogorov-Smirnov type tests for the 47 high and 22 low redshift (Fit C) supernovae from Perlmutter et al. (1999). The lowest p-value is 0.16, so there are no significant differences.

We have not found any significant differences between the low and high redshift samples. Of course this does not mean there are none. There may be a model of evolution which fits the data better than no evolution. Our results show that we are not forced by the current data to conclude any change in Type Ia supernovae properties.

7. DISCUSSION

We have made an extensive comparison of the different light curve fitting procedures that have been used in estimating cosmological parameters, and tried to determine where the major differences come from. It is important to remember that using different methods and templates to model the same light curve data is like using different equations on the same measurements. Different results will be obtained if different models are used. We have studied the differences between the methods in order to improve our understanding of what aspects of supernovae the different methods are sensitive to.

We have found that for supernovae without observations before maximum light there can be big differences in results between the MLCS method and the other two, with the MLCS method measures of light curve width exhibiting a double-valued relationship with the Stretch and Dm15 corrections. Since the template light curves are different, the supernovae most different from a standard supernova show the largest differences in distance modulus. This is not a problem for well-sampled light curves however. The Stretch method is not sensitive to the existence or not of data points before maximum light. Of course the MLCS method is evolving. As subsequent versions are published it will be important to see if they are sensitive to the sampling of the light curves.

It is interesting to note how well the slopes of the width-brightness relations used by the Dm15 and Stretch methods agree, since that used in the Dm15 method ($\alpha \approx 1.6$) was determined using supernovae with redshifts between 0.01 and 0.1, while the Stretch result ($\alpha = 1.7$) came from the whole sample over the range 0.01 - 0.97. Using the SCP supernovae in addition to the other groups we have used in this paper, the Stretch method obtains $\alpha = 1.8$, still in good agreement. All these numbers are for host extincion corrected measurements. There is no way to test if the relation is the same for high and low redshift supernovae in the context of light curve fitting only.

In the comparison of the various light curve fit parameters and their differences between analysis methods for low and high redshift data, we have found no evidence for differences in dependencies between the various parameters at low and high redshift. Note that our tests cannot detect an arbitrary redshift-dependent change in distance modulus, such as Drell, Loredo, & Wasserman (2000) propose. Simple functions of redshift can be ruled out using observations at higher redshift (Aldering, 1998; Riess et al., 2001). More complicated redshift-dependencies cannot be tested using distance moduli and redshift alone (since any $\mu - z$ relation can be fitted if arbitrary functions are allowed) but these are not physicsally motivated models.

Most realistic ideas about evolution would not predict a dependence only on redshift. We would expect a more complicated dependence on other properties of the supernova, analagous to the dependency of the absolute magnitude on light curve width, which might change the correlations between various parameters at different redshift. In addition, evolution has been tested using supernovae risetimes (Aldering et al., 2000) and spectra (Perlmutter et al., 1998; Coil et al., 2000). For the intergalactic dust considered in Aguirre (1999), some reddening is expected, so all the light curve fit parameters should be affected. These are the motivations behind the tests we have done here. As the amount and quality of data improves, we can expect to be able to test specific ideas of how the properties of Type Ia supernovae change in different environments.

ACKNOWLEDGEMENTS

We would like to thank Andreas Albrecht for helpful discussions. The observations described in this paper were primarily obtained as visiting/guest astronomers at the Cerro Tololo Inter-American Observatory 4-meter telescope, operated by the National Optical Astronomy Observatory under contract to the National Science Foundation; the Keck I and II 10-m telescopes of the California Association for Research in Astronomy; the Wisconsin-Indiana-Yale-NOAO (WIYN) telescope; the European Southern Observatory 3.6-meter telescope; the Isaac Newton and William Herschel Telescopes, operated by the Royal Greenwich Observatory at the Spanish Observatorio del Roque de los Muchachos of the Instituto de Astrofisica de Canarias; the Hubble Space Telescope, and the Nordic Optical 2.5-meter telescope. We thank the dedicated staff of these observatories for their excellent assistance in pursuit of this project. In particular, Dianne Harmer, Paul Smith and Daryl Willmarth were extraordinarily helpful as the WIYN queue observers. We thank Gary Bernstein and Tony Tyson for developing and supporting the Big Throughput Camera at the CTIO 4-meter; this wide-field camera was important in the discovery of many of the high-redshift supernovae. This work was supported in part by PPARC, UC Davis, the Physics Division, E. O. Lawrence Berkeley National Laboratory of the U. S. Department of Energy under Contract No. DE-AC03-76SF000098, and by the National Science Foundation's Center for Particle Astrophysics, University of California, Berkeley under grant No. ADT-88909616.

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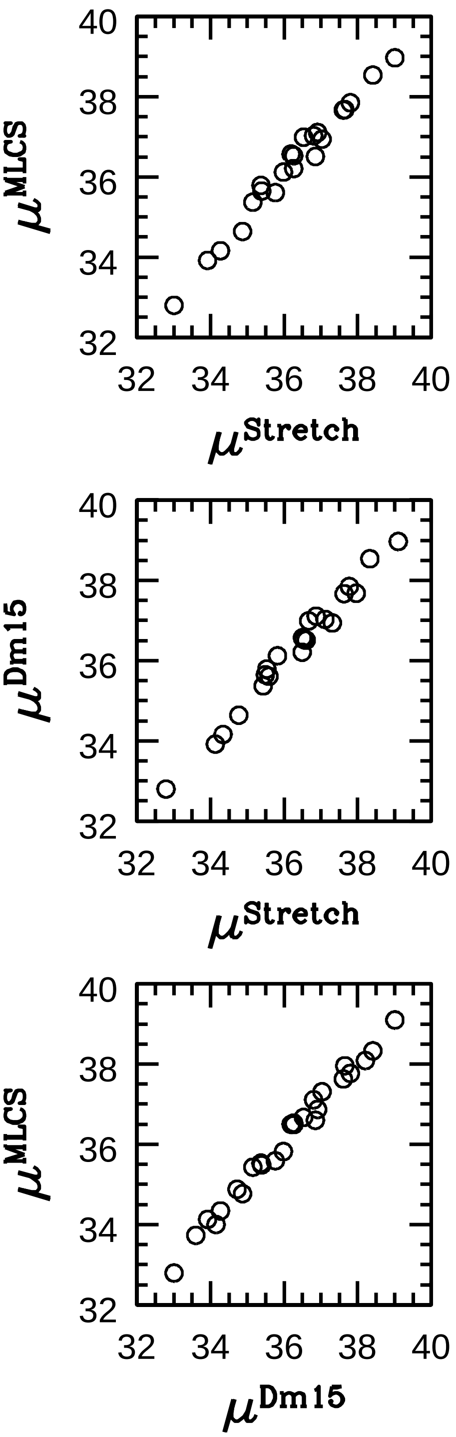

FIG. 1.— Distance moduli found for the low redshift supernovae by the three different methods, using Burstein and Heiles extinction for our galaxy, and no host extinction correction for the Stretch results. Distance scales are arbitrary, but consistent.

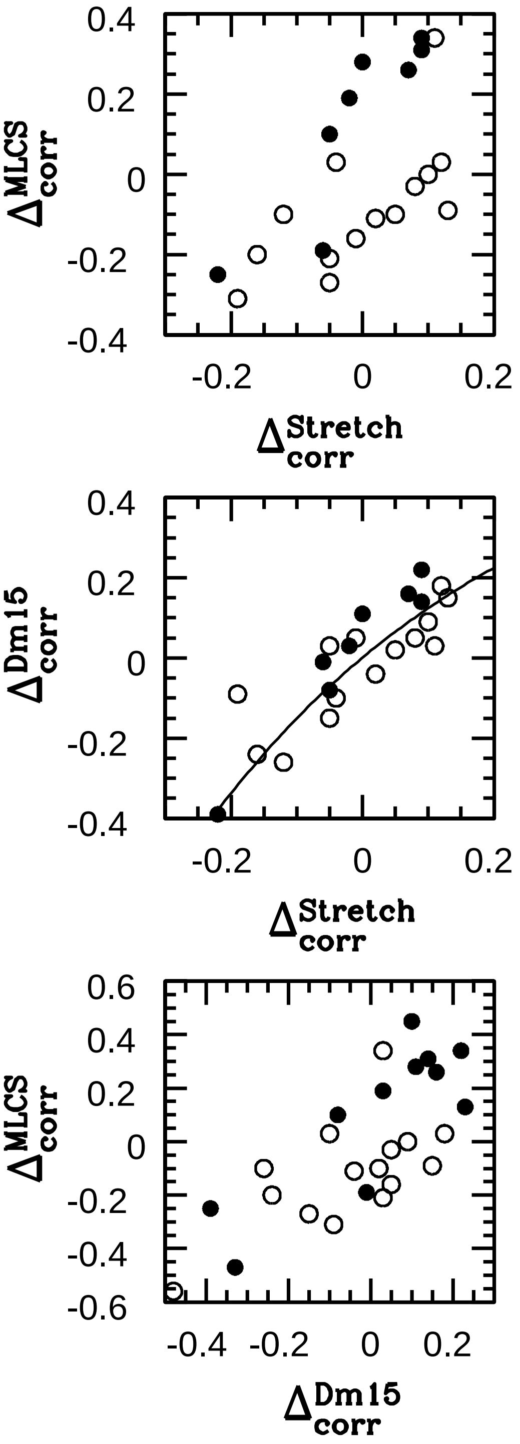

FIG. 2.— Magnitude correction found for the low redshift supernovae by the three different methods. The magnitude corrections have been shifted by amounts that make them agree on average, as any offset between them is merely a difference in choice of standard supernova. The solid curve on the Dm15 - Stretch plot is the prediction for the relation between the magnitude corrections using the different width-brightness relations used by the two methods (see Section 5.1). The filled circles are the supernovae with observations before maximum light (see Section 5.2 for discussion.)

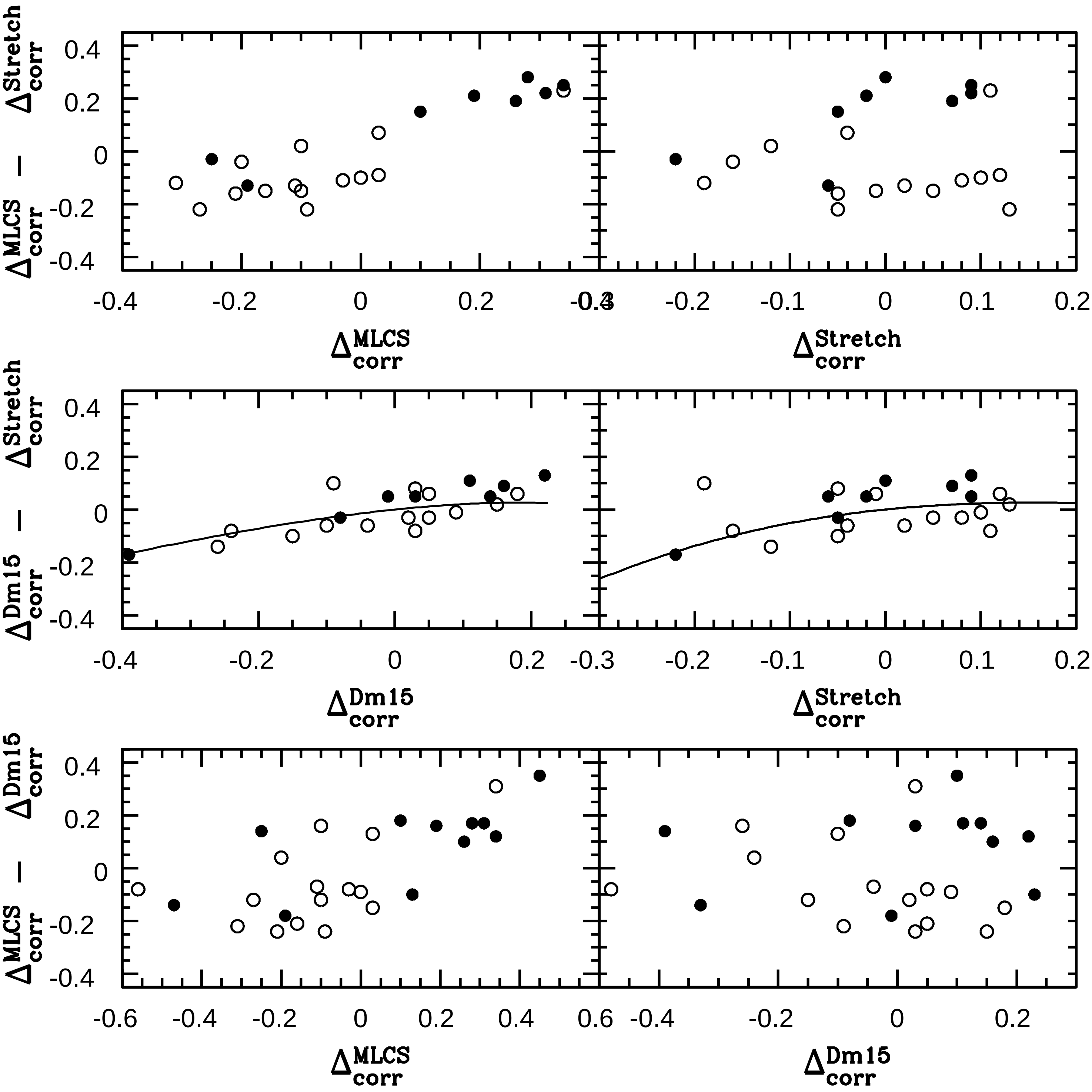

FIG. 3.— Difference in magnitude correction as a function of magnitude correction as measured by the various analysis methods. The average has been subtracted out, as this is merely a difference in choice of standard supernova. The y-axes for all pairs of methods are the same so that the overall level of agreement can be compared. The solid curve on the Dm15 - Stretch plots is the prediction for the relation between the magnitude corrections using the different width-brightness relations used by the two methods (see Section 5.1). The filled circles are the supernovae with observations before maximum light (see Section 5.2 for discussion.)

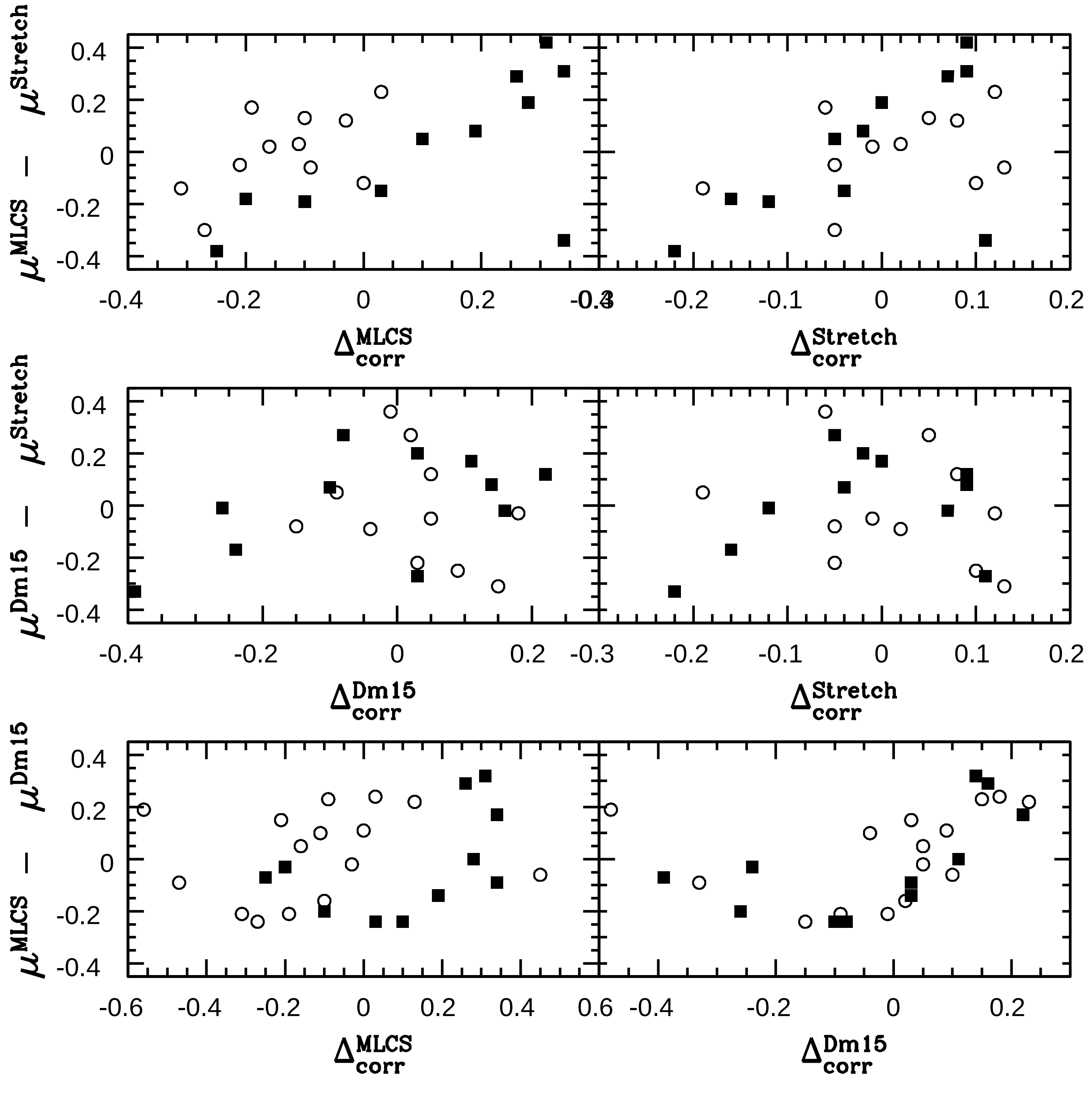

FIG. 4.— Difference in distance moduli as a function of magnitude correction as measured by the various analysis methods. The average has been subtracted out, as offsets from zero can be accounted for by the fact that the Stretch values are not corrected for host extinction, and that the methods have used different distance scales. The magnitude corrections have also been shifted by amounts that make them agree on average, as any offset between them simply means different absolute magnitudes are being used for a standard supernova. The axes for all the graphs are the same as the corresponding ones for the difference in distance moduli in Figure 3. The filled squares are the supernovae in Group II, defined by the relation $\Delta^{MLCS}_{corr} \sim 2\Delta^{Stretch}_{corr}$. The "odd" supernova in Group II is one with large host extinction. See Section 4.2 for discussion.

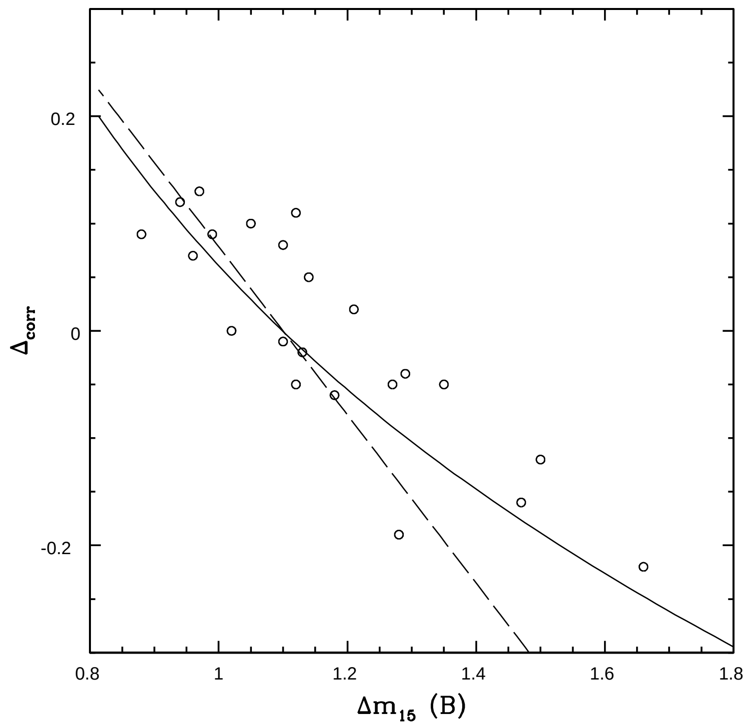

FIG. 5.— Possible different formulae for the magnitude correction as a function of Δm_{15}. The straight (dashed) line is the linear Dm15 magnitude correction $-0.784(\Delta m_{15} - 1.1)$ as used in Hamuy et al. (1996b) and the solid curve is the Stretch magnitude correction $\alpha(s-1)$ using the non-linear relationship between Δm_{15} and stretch s. The data is also shown, plotted as $\Delta_{corr}^{Stretch}$ (found by Stretch) versus Δm_{15} (found by Dm15).

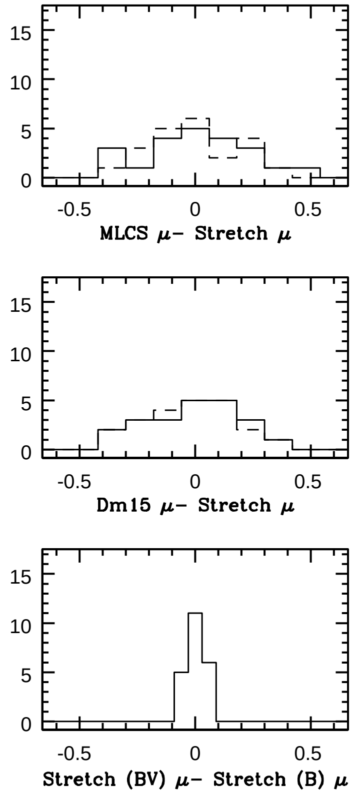

FIG. 6.— The upper plot shows histograms of the difference in distance moduli between MLCS and Stretch for Stretch results found using B band only (solid line) and those found using both B and V bands (dashed line). The second plot shows the same comparisons for the Dm15 and Stretch methods. The third plot shows the distance moduli differences between the Stretch (B) results and Stretch (BV) results. The Stretch results using both B and V bands are much closer to the Stretch results using only the B band than the MLCS or Dm15 results.

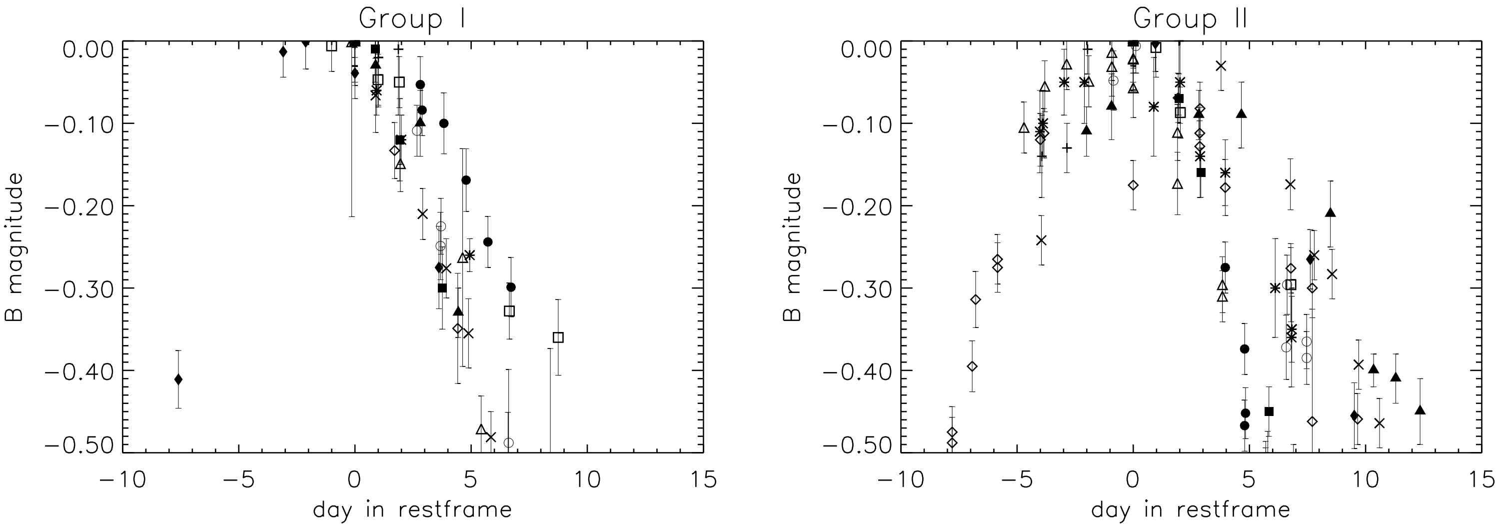

FIG. 7.— Light curve B band data in the rest-frame for Groups I and II. Only one supernova in Group I has observations before maximum light, while most supernovae in Group II have observations before maximum light, making it easier to define the time of maximum.

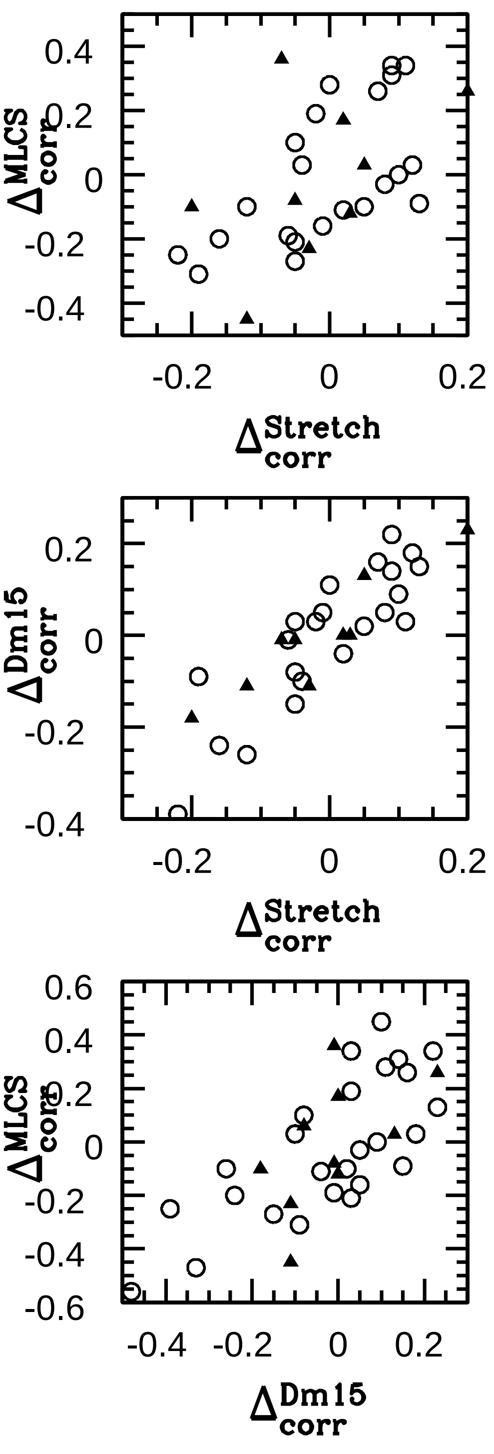

FIG. 8.— Magnitude correction found for the whole sample supernovae by the three different methods. The open circles are the low redshift supernovae and the filled triangles are the high redshift supernovae.

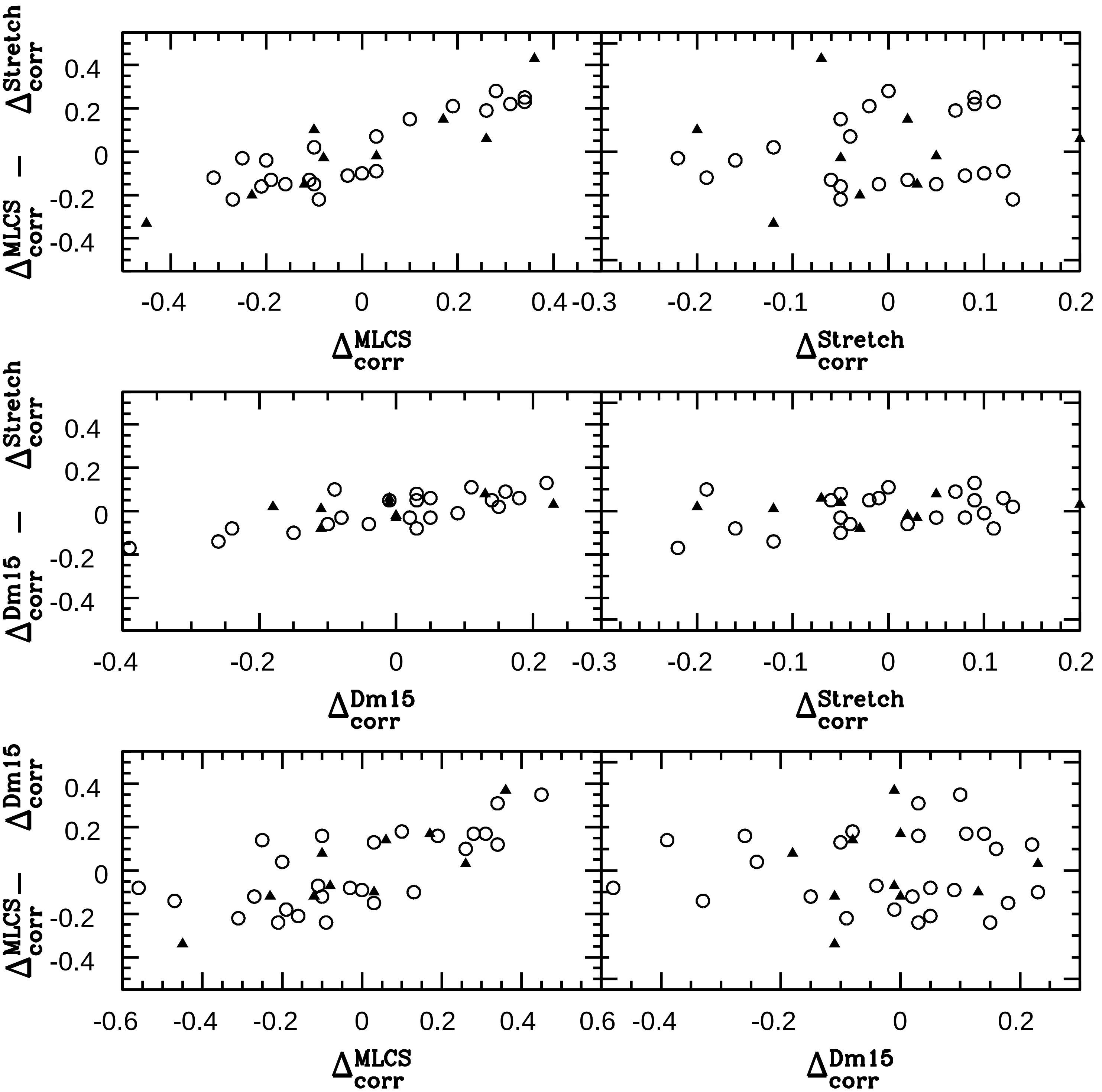

FIG. 9.— Difference in magnitude correction as a function of magnitude correction for all supernovae. The open circles are the low redshift supernovae and the filled triangles are the high redshift supernovae.

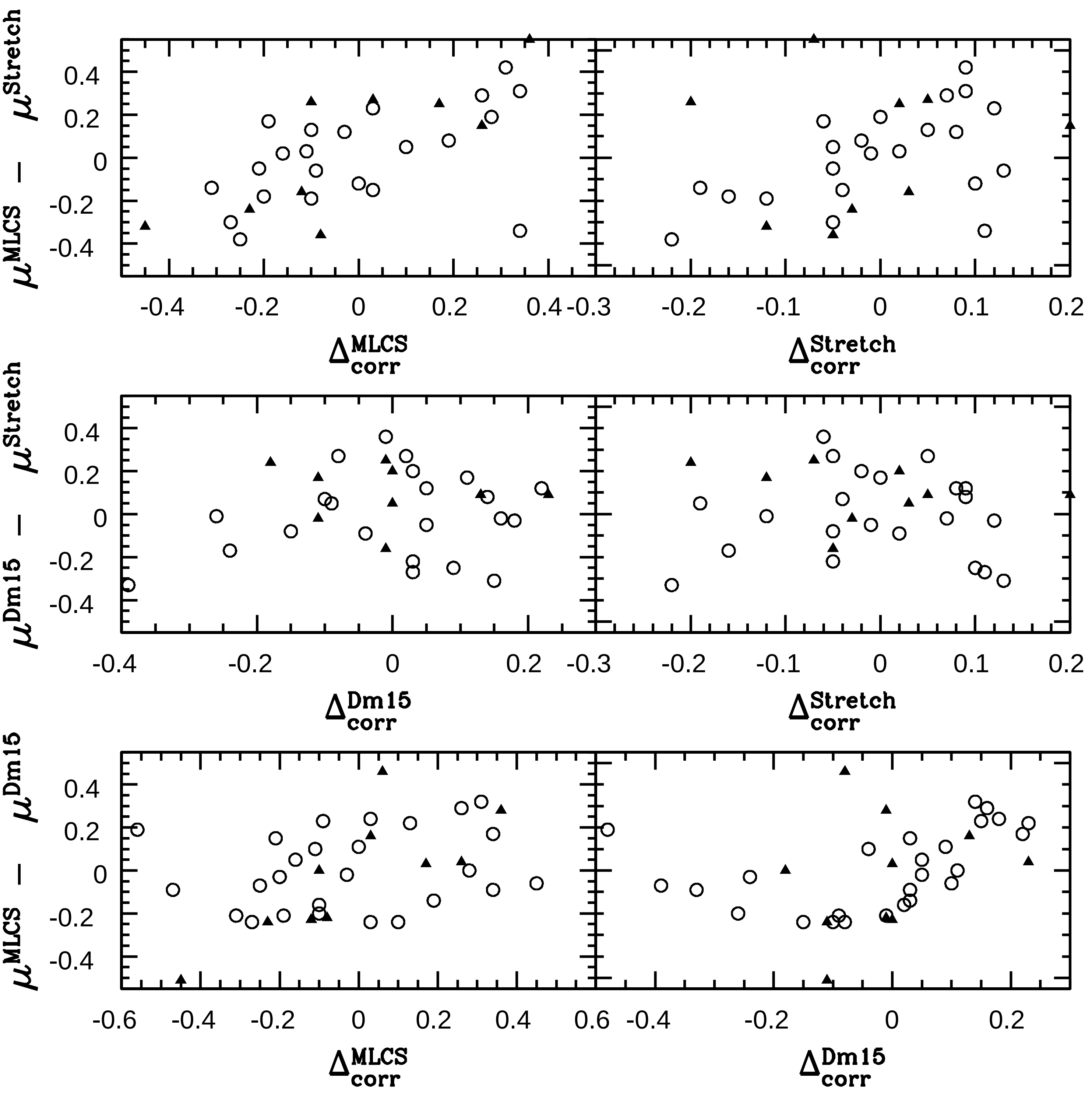

FIG. 10.— Difference in distance moduli as a function of magnitude correction. The open circles are the low redshift supernovae and the filled triangles are the high redshift supernovae.