A Unified Lightcurve Fitting Algorithm for Type Ia Supernovae

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SN Ia Lightcurve Fit

ABSTRACT

I describe a new algorithm for modeling SN Ia lightcurves which can reduce significantly fitting errors and can be generalized to achieve accurate fits for lightcurves obtained at any color. It can be applied to derive quantities that can be used for distance estimates using SN Ia supernova as probes of cosmological parameters.

Subject headings: supernova: general, distance scale

1. Lightcurves of Type Ia Supernovae

Lightcurves of SN Ia are found to be powerful tools for deriving cosmological distances. When coupled to decline rate (Pskovskii 1977; Phillips 1993), or modeled by template fitting (Riess et al. 1996), or corrected by stretches of the time coordinates (Perlmutter et al. 1997), they provide distance estimates accurate to around $0^m.1$-$0^m.18$ (e. g., Phillips et al. 1999). SN Ia were used to derive cosmological parameters and it was found that a non-zero cosmological parameter (or dark energy) is required to explain the apparent deceleration of the universe at redshift around 0.5 (Riess et al. 1998; Perlmutter et al. 1999).

Existing lightcurve fitting procedures, however, often fail in making precise fits to the observed data points. In many cases the fits apparently systematically miss the observed data points around optical maxima and various other stages of the lightcurves. In several cases such as SN 2000cx (Li et al. 2001) the fits provide no reasonable description to the data at all.

The purpose of this *letter* is to present a unified lightcurve fitting algorithm (ULFA) which can accurately fit SN Ia lightcurves to within observational errors.

2. Mathematical Model

Perlmutter et al. (1997) showed that the B and V lightcurves of a Type Ia supernova can be fitted by template lightcurves at corresponding filters by stretching the time coordinates. Stretch fit is a linear transformation in time coordinate and it is known that it does not apply to data close to and after nebular phase. This transformation can be generalized by allowing the time transformation to be nonlinear. Experimenting with available lightcurves published by Hamuy et al. (1996) and Riess et al. (1998), I found the following equation provides excellent fits to practically all of those supernovae,

$$\tau_n(t) \;=\; t + t_0(1 - e^{-t^2/2\tau_0^2})t/|t|, \tag{1}$$

where τ_n is the transformed time coordinate, t the redshift corrected original time, t_0 and τ_0 parameters to be determined from the lightcurve fits. In general, $d\tau_n/dt \;>\; 0$ is required to ensure one to one transformations of time. For equation (1), this requirement leads to the constraint $-\tau_0 e^{1/2} \;<\; t_0 \;<\; \tau_0 e^{1/2}$. Other forms of functions similar to the one above can be constructed. I have tested the procedure using

$$\tau_n(t) \;=\; t \;+\; t_0\, tanh(t/\tau_0) \tag{2}$$

where $tanh$ is the hyperbolic tangent function, and

$$\begin{aligned} \tau_n(t) \;&=\; (1 + t_0/\tau_0)\, t \qquad && \text{for } |\mathrm{t} - \tau_0| \;<\; 0 \\ \tau_n(t) \;&=\; t_0 + \tau_0 && \text{for } |\mathrm{t} - \tau_0| \;\leq\; 0. \end{aligned} \tag{3}$$

Equation (3) is similar to the conventional stretch method but the stretch only applies between day $-\tau_0$ and τ_0. These functions are plotted in Figure 1. For both equations (2) and (3), the results are comparable to the ones given by equation (1), and differ significantly only for a few peculiar cases. The above functional forms can in fact be constructed from well observed supernovae, more high quality supernova light curves are needed for this purpose.

With the above time transformation, and using the B and V templates constructed by Riess et al. (1999) (modified to ensure smoothness of second order derivatives and extrapolations to allow for more time coverages pre-max and the nebular phase), significant improvements are achieved for the fits to all of the B and V lightcurves of supernovae in the samples of Hamuy et al. (1996) and Riess et al. (1998). The fits are equally good for data points at all epochs.

Although the above prescription works perfectly well for the B and V lightcurves, it does not provide as good fits to R band data, and fails in most cases for I band data. A new method must still be constructed for modeling the lightcurves in these colors.

To achieve this goal, I constructed a multi-component model to the observed lightcurve. In the simplest case of a two component fit, the second component can just be a time delayed "echo bump" of the first component, *i. e.*

$$\begin{aligned} f_{obs}(t) &= \alpha\, [f_T(t - t_{max} + \tau_n(t - t_{max})) + \\ &\qquad \delta\, f_T(t - t_{max} - t_d + \tau_n(t - t_{max} - t_d))], \end{aligned} \tag{4}$$

where $f_{obs}(t)$ is the observed magnitudes converted into flux, f_T the template magnitudes converted into flux, t_{max} time at maximum, τ_n the transformed time coordinates as given in equation (1), or in any other parameterized forms to be determined by better data, t_d a parameter measuring the time delay of the second component, α and δ two parameters that provide the flux levels of the supernova and the relative contributions of the second component. Equation (2) has four parameters characterizing a supernovae lightcurve. Modeling well observed lightcurves should determine possible ranges and correlations among them. Inverting equation (2) should also provide more realistic lightcurve templates.

I have applied equation (2) to all the available supernova lightcurves and found that satisfactory fits can always be achieved for data before day 60 by applying the V lightcurve to *all* lightcurves in *all* colors. At very late phases, the B and V light curves depends linearly with time and is easy to model either with an additional coefficients on equations (1), (2), and (3), or by constructing template light curves for each individual filter.

3. Examples of the Fits

The template used in these examples is a slightly modified version of the one given in Riess et al. (1996). Some of the four parameters characterizing equations (1) and (2) can actually be determined from observations, and are not really free parameters. I have left all four parameters free in these examples, however.

Normal supernovae are all fitted well by this method. I show here only cases which are considered *peculiar* and existing fitting procedures fail to provide good fits. Figure 1 shows the fits to SN 1995bd. The data are taken from Riess et al. (1999). The MLCS fit to this supernova can be found in Figure 3 of Riess et al. (1996), where it is clearly seen that there are significant systematic deviations of the data points from the fitted curves. A uniform time stretch provides poor fits even to data around maximum. In particular, the observed peak in the B-band was not well fit (as is also true for many other supernovae shown in Figure 3 of Riess et al. (1996)) which can cause severe errors in determining color around maximum. The ULFA fits shown in Figure 1 uses only the V band template and the χ^2 per degree of freedom (dof) are around 1 for data in all four filters (B, V, R, and I). The parameters used in the fits are shown in Table 1.

I show also fits to the sub-luminous SN 1999by in Figure 2. The data are from Garnavich et al. (2001). Both fits to B and V give χ^2 per degree of freedom below one. The

Table 1. Examples of SN Ia Light Curve Fits

Filter	t_{max} [a]	M_{max}	δ	t_0	τ_0	t_d	χ^2/dof
			SN 1995bd				
B	0086.330	17.309	-0.060	-6.432	13.604	36.221	1.118
V	0087.773	16.489	0.107	-6.675	18.650	24.531	0.613
R	0087.330	16.029	0.232	-6.471	16.240	27.767	2.002
I	0085.453	16.026	0.411	-1.373	8.497	30.772	1.915
			SN 1999by				
B	1308.843	13.661	-1.525	-18.632	18.607	6.488	0.256
V	1310.869	13.135	-0.409	-1.322	3.084	6.961	0.395
			SN 2001el				
J	2179.630	13.076	0.327	-42.278	41.244	32.271	22.125
H	2181.102	13.076	0.781	6.388	58.723	26.037	2.287
K	2180.877	12.878	0.702	-13.430	31.456	26.535	1.522

[a]In JD-2,450,000

observations have poor coverage at around day 25 to constrain the second component in the R and I band. The fits to R and I are consistent with no significant secondary component.

ULFA can also be applied in a consistent way to J, H, and K band. As examples I show such fits for SN 2001el in Figure 3. The fits are not as good as those for the other filters at shorter wavelengths, but there is ample room for improvement in both the templates and the time transformation formula.

4. Conclusions and Discussions

I have developed a robust algorithm that can be used to fit observed SN Ia lightcurves at various colors. The algorithm has yet to be finalized by more thorough analysis of existing data and by more high quality observations in all wavelengths. More details will be given in a subsequent paper which will study the correlations between the parameters and their consequences on distance calibrations.

Future attempts should focus on constructing the correct time transformation equations and the template light curves on different filters. These equations and templates should also be combined with the constraints derived from the CMAGIC method (Wang 2002) to deduce simulaneously color-excess due to dust extinction, the light curve properties, and the magnitudes required for distance calibrations.

The code used in these fits can be obtained by contacting the author.

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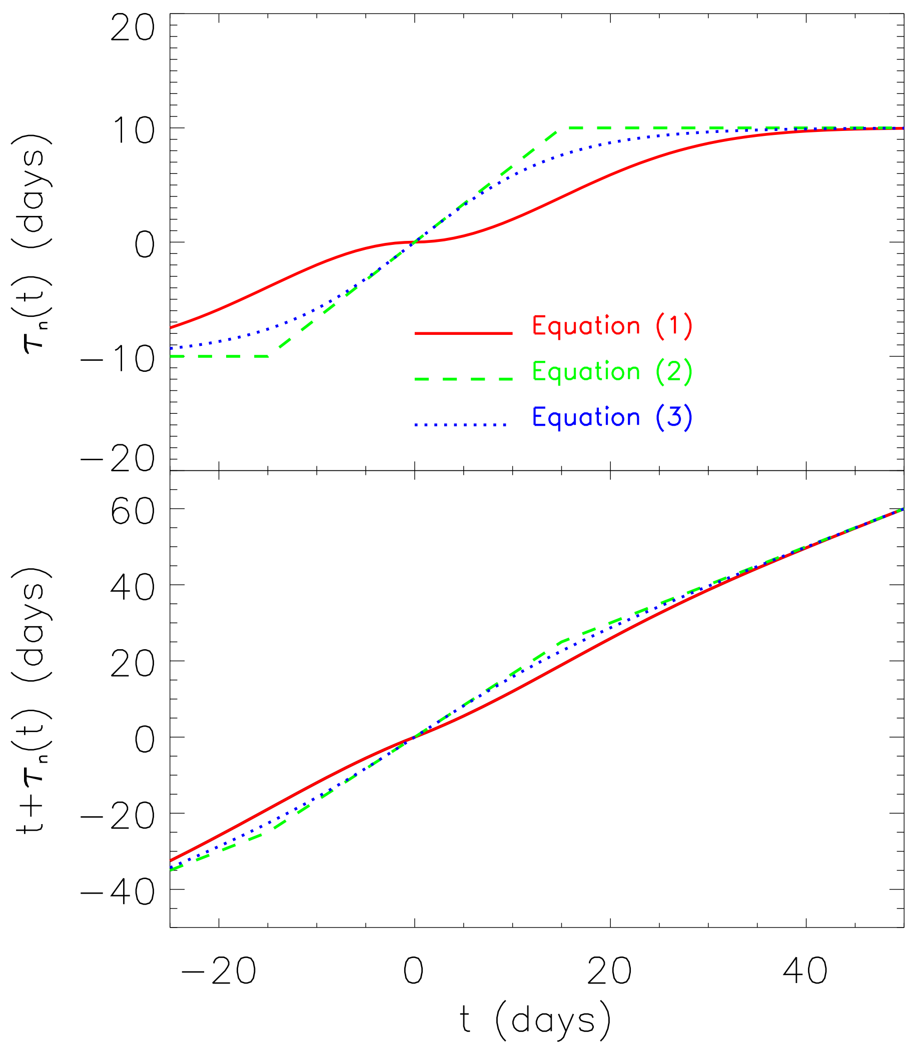
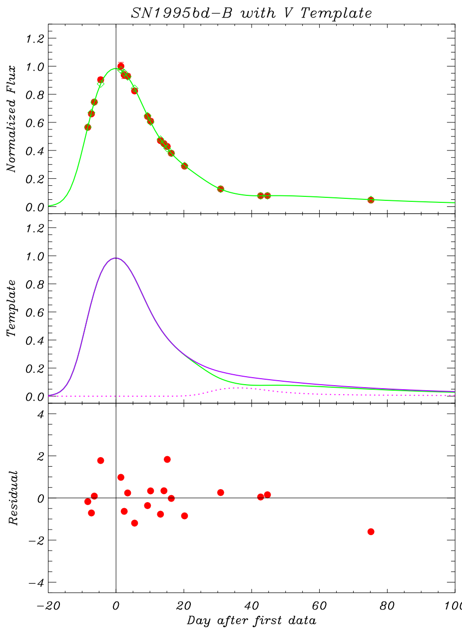


Fig. 1.— Time transformation functions for $t_0 = 10$ and $\tau_0 = 15$. The top panel shows the difference between the new coordinate and the original time coordinate. The lower panel shows the transformed time versus the original time. The curves are for equations (1) (red solid line), (2) (blue dotted line), and (3) (green dashed line).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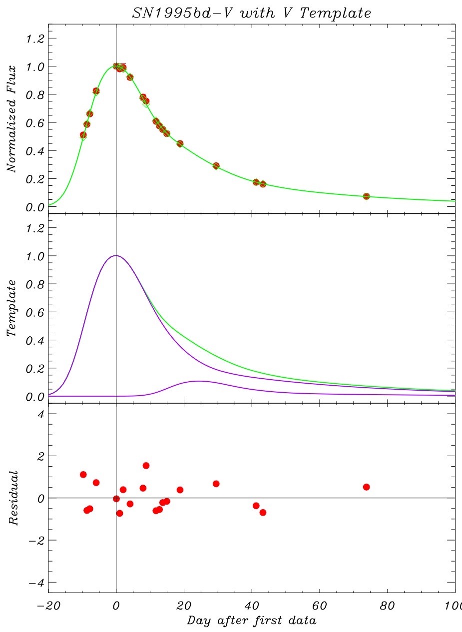
Normalized Flux
Template
Residual
Day after first data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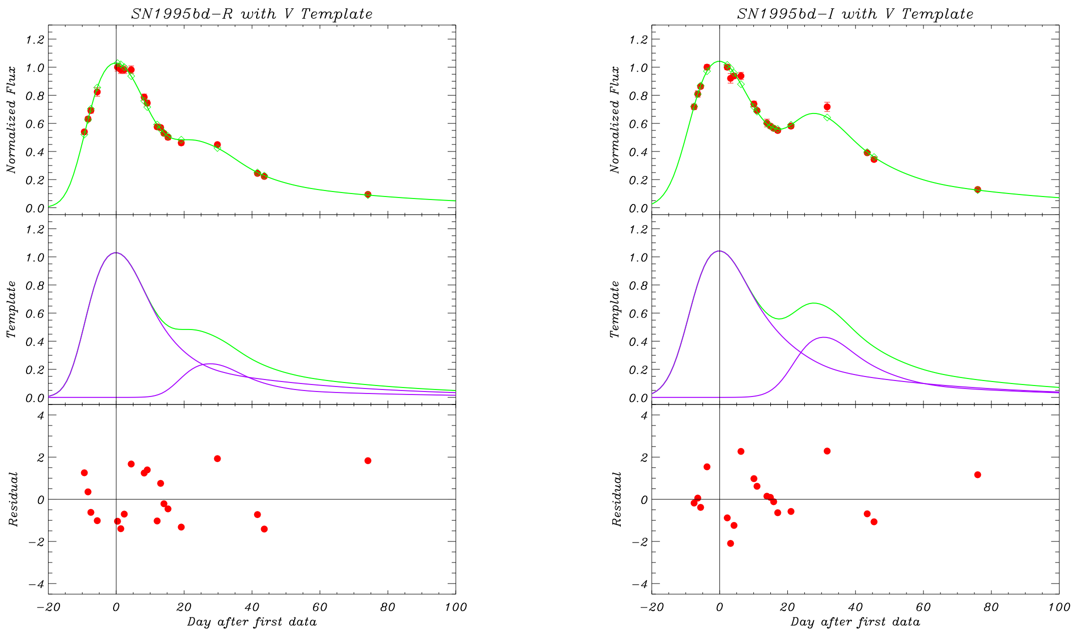


Fig. 3.— ULFA fits of SN 1995bd. All of the fits were performed using a V-band template. (a), (b), (c), and (d) are for data of B, V, R, and I filters. The top panel shows the observed data (solid dots) with the fitted curve over-plotted. The vertical axis is in flux scale and is normalize to the peak flux. The middle panel shows the absolute values of the various components of the fits. The un-delayed and delayed (first and second terms in equation (2)) are shown in purple and pink lines respectively. The green lines show the composite lightcurves identical to the green lines in the top panels. For (a) the second component is shown in dashed line and is actually negative. The lower panels show the deviations from the fitted curves divided by corresponding errors.

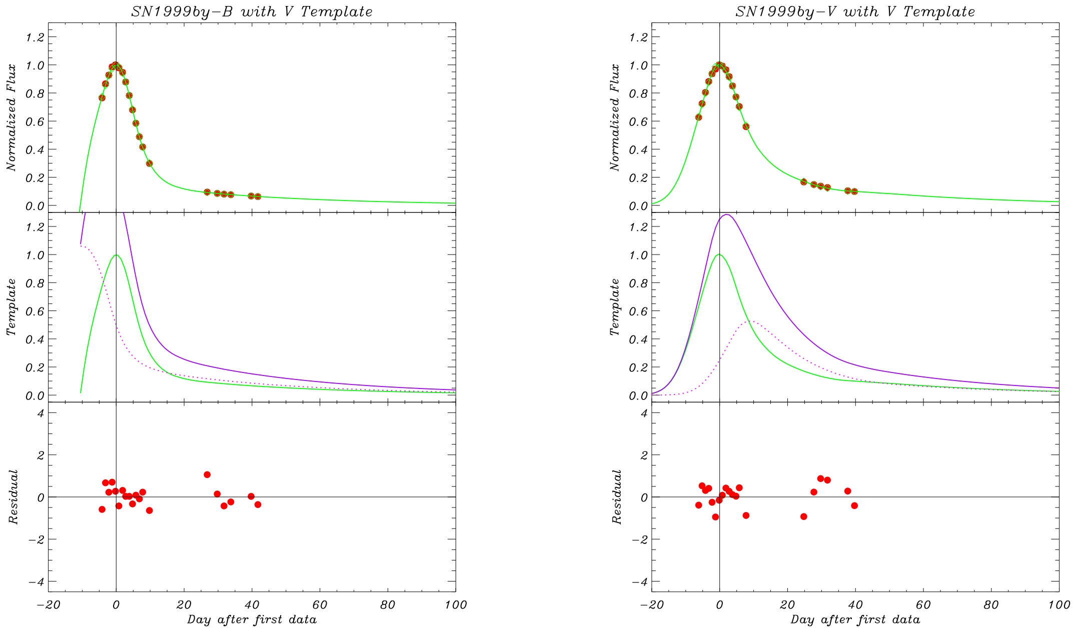
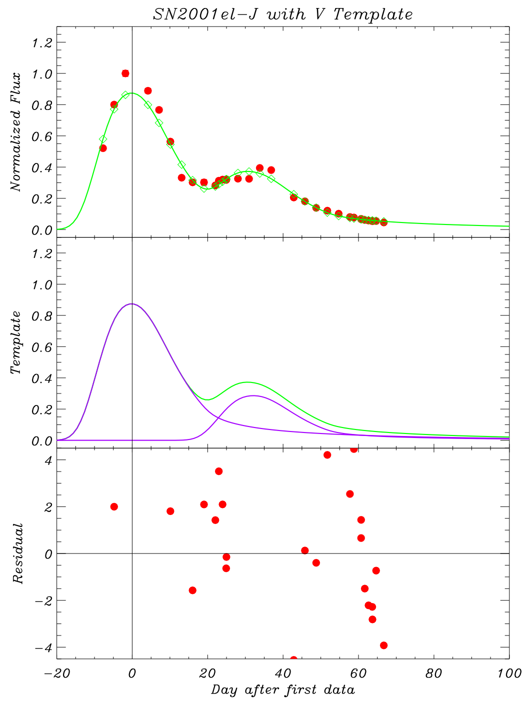


Fig. 2.— Same as Figure 1, but for the B and V band of the sub-luminous supernova SN 1999by.


SN2001el–J with V Template
Normalized Flux
Template
Residual
Day after first data

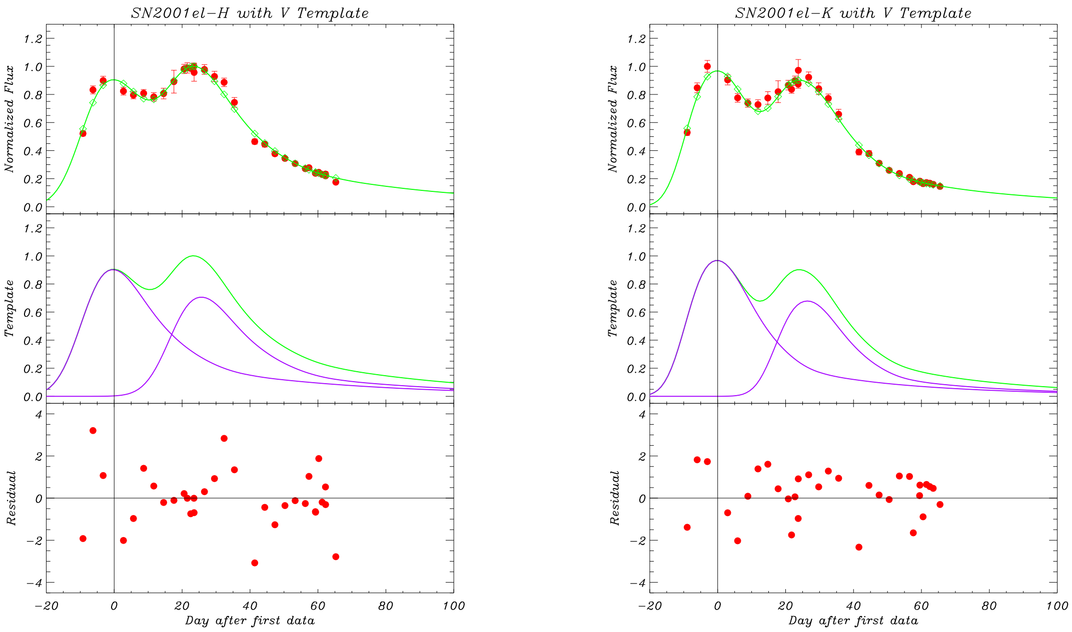


Fig. 3.— Same as Figure 1, but for the J, H, and K band data of SN 2001el.

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