Why are Supernovae at $z > 1$ Needed to Probe Dark Energy?

A Pictorial Primer

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In a purely theoretic sense, the dynamical influence of dark energy rapidly decreases above redshifts $z \approx 0.6$. Moreover, an idealized Fisher matrix calculation shows that the "sweet spot" of sensitivity to the equation of state w lies at $z \approx 0.3$. So why are observations at $z \geq 1$ necessary for characterizing the dark energy? The answer lies in the breakdown of the ideal case:

- Systematic errors
- Cosmological degeneracies
- Dark energy model degeneracies

The required survey depth depends on the rigor of our scientific investigation, how much we are willing to assume about the other parameters entering into the determination of the dark energy equation of state. A precision estimate of w could in principle be carried out with an experiment observing supernovae only out to $z \approx 0.3$. But the accuracy of this value would be uncertain, subject to bias from breakdown of assumptions on other, degenerate or congeneric parameters. (*Congeneric* – resembling in nature or action – also has the connotation of a molecule that acts analogously but yields a quite different taste).

First we address the shibboleths of the sweet spot and the dynamic influence range. Both are correct in a formal sense, but fail in reality due to the weakness of their underlying assumptions. The sweet spot derives from the Fisher matrix in a process that relies on parabolic (gaussian) behavior of the likelihood surface around the fiducial parameter set. This holds only in the absence of correlated errors (e.g. systematics) and an exactly known matter density Ω_m. So the precision determination of $w(z = 0.3)$, say, has only limited meaning and limited accuracy. See Figure 1 for an illustration of the role of systematics alone, with no degeneracy contribution from the cosmology or dark energy model (i.e. fixed Ω_m, constant w). Plus the sweet spot has little leverage on the value of w at other redshifts, i.e. the time evolution w_1.

The dynamical influence argument relies on a putative measurement of the dark energy density Ω_w at a specific redshift, in comparison to the matter contribution, achieving equality at z_{eq}. The issue of the changeover from accelerating to decelerating expansion is analogous. But the integral nature of the dependence of the dynamical relations (e.g. distances) on the parameters prevents this from being measured directly. Even a tightly controlled tomographic experiment yields only $H(z)$ directly, still involving an integration over $w(z)$. This causes an inertia that stretches the importance of the dark energy to higher redshifts. For example, say the total dynamical equation of state (which itself includes both $w(z)$ and its integral) crosses from less than to greater than $-1/3$ at some z_{ac}, the mark of the accelerating-decelerating transition. This does not show up in a direct observable like the turnover of the magnitude-redshift diagram until perhaps a redshift $2z_{ac}$.

To see evidence of the accelerating-decelerating transition, a key discriminator from generically monotonic systematic effects, we are forced to redshifts $z > 1$, even though the transition takes place at much lower redshifts. Figure 2 explicitly demonstrates the dynamical influence of dark energy at redshifts much higher than the formal equality or acceleration transition values.

Similarly, to disentangle nonmonotonic biases such as a different value of Ω_m one is driven to $z > 1$. While the best hope is accurate complementary determination by a probe insensitive to or depending differently on the dark matter properties, given some remaining finite uncertainty one must strive for as long a redshift baseline as possible to maximize the distinction between the equations of state required to fit the data. Even with a pinned down value of Ω_m, our ability to distinguish dark energy models improves with a greater redshift window as very different scalar field potentials, and hence physics, can still yield similar magnitude-redshift relations over a finite range. This is especially exacerbated at low redshifts. For example the time, or redshift, variation in the equation of state, e.g. w_1, is poorly determined for low redshift experiments, with a steep error dependence at $z < 1$. Figure 3 illustrates these confusion possibilities from cosmological and dark energy model degeneracies.

What in principle looked easy in an idealized case become more difficult, but still well within reach, in a realistic treatment. The following graphs show in more detail how the redshift range required to attain a certain parameter accuracy changes as one discards assumptions in turn. Figures 4-6 demonstrate the influence of relaxing the assumptions of a perfectly known Ω_m and no systematics for three different survey depths. The model for the systematic error is $\sigma_m = 0.02\,(1.7/z_{max})\,(1+z)/2.7$, irreducible over a 0.1 z bin. The main features of this ansatz is an error increasing linearly in z (or $1+z$), not vanishing as $z \to 0$, and larger at a given z for a shallower survey.

The main effect of the systematic is to extend the contours in the $w_0 - w_1$ plane along the major axis, i.e. degrading the estimates of both parameters. Uncertainty in Ω_m tends to fatten the contours, but keep them "kissing" to the fixed Ω_m case. Note this implies that sometimes a mere quotation of the limiting errors would not turn up the full degradation: for example in the $z_{max} = 0.5$ or 0.9 cases the limits on w_0 or w_1 change relatively little by increasing the uncertainty in Ω_m, but the area of the error regions increase by up to a factor of three as a new degeneracy direction enters. So one must be cautious in dealing with simple quotes such as "this determines w_0 to ± 0.07."

Figure 7 shows the best and worst case contours for each survey depth: those with both ideal assumptions valid, and with both invalid. Here one clearly sees several important properties:

1. w_1: A shallow survey is incapable of appreciably limiting w_1, even for perfect assumptions; a medium survey fails when the assumptions are relaxed.

2. Depth: While there appears to be relatively little difference between the results of a $z_{max} = 0.9$ and 1.7 survey under the ideal case, e.g. estimation of w_0, w_1 relaxes by 0.02, 0.13, for the imperfect case the 1σ constraints degrade by 0.09, 0.34. At higher z the leverage on systematics improves such that the main influence on the degradation is the degeneracy from an uncertain Ω_m.

3. Like to like: Experiments should be compared under the appropriate assumptions. An idealized $z = 0.9$ survey claims limits on w_0, w_1 *better* by 0.01, 0.07 than the imperfect $z = 1.7$ one, in constrast to the above like to like comparison.

On another level, one could compare SNAP, say, with surveys with different depths and systematics to obtain a clear view of their relative impact, within the ansatz presented here. For example one could place constraints on the amplitude and slope of systematics such that the results fell within a certain percent of SNAP's. In the quantitative limit one would of course want a full model of each survey for comparison. But more qualitatively – and in a less model dependent manner – the discussion in the first part of this note and the plots in the second part show that naïve reliance on arguments of the sweet spot and density or acceleration transitions prove insufficient and misleading for understanding how to probe the dark energy.

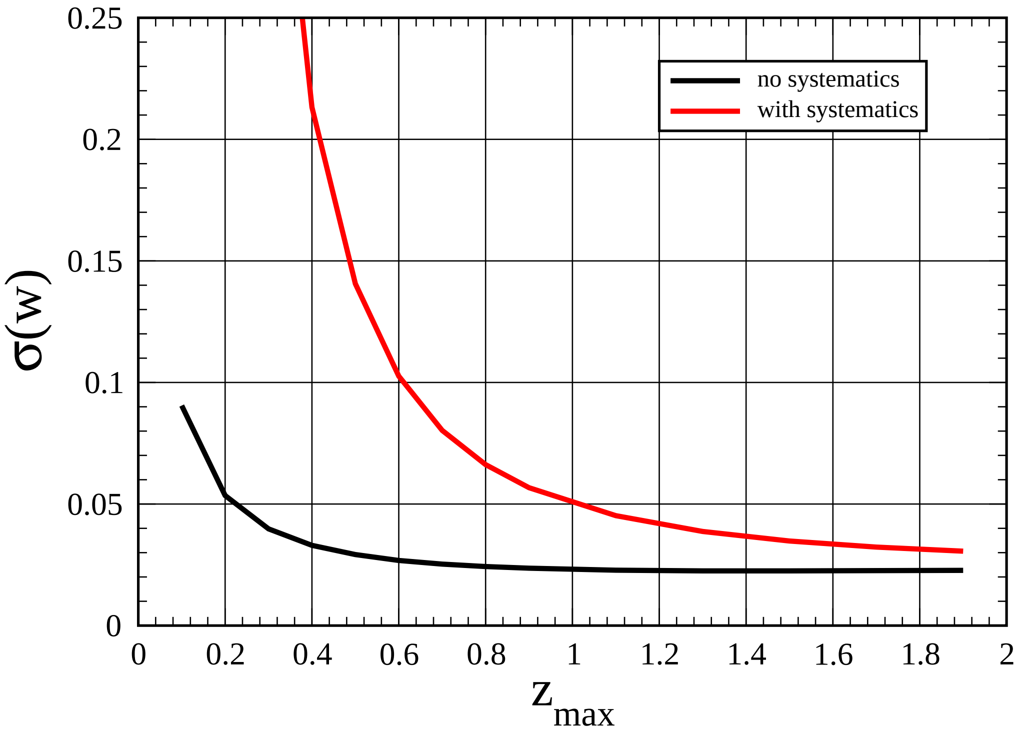

Figure 1: The gaussian error estimated in a constant equation of state *w* vs. survey depth. In an idealized view one improves the errors almost to the asymptote by the sweet spot at $z \approx 0.3$. However addition of a systematic error drastically changes the situation, even keeping ideal knowledge about the cosmology and dark energy model.

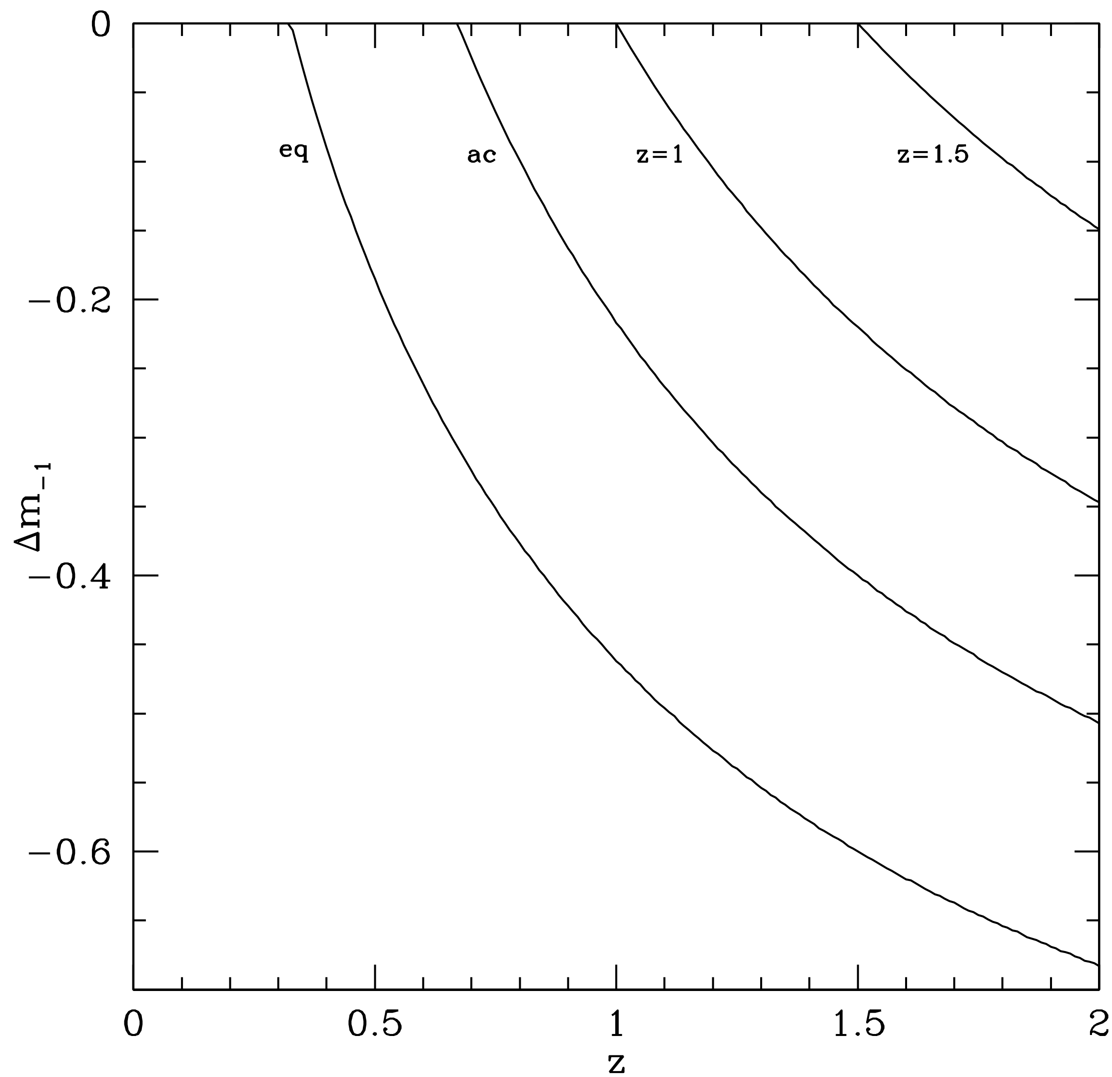

Figure 2: Dynamical influence of dark energy persists substantially beyond the redshifts of equality or the acceleration-deceleration transition. The curves show the magnitude difference from a cosmological constant model when the dark energy is ignored (treated as ordinary matter) above different redshifts. Detectable influence remains even at $5z_{eq}$.

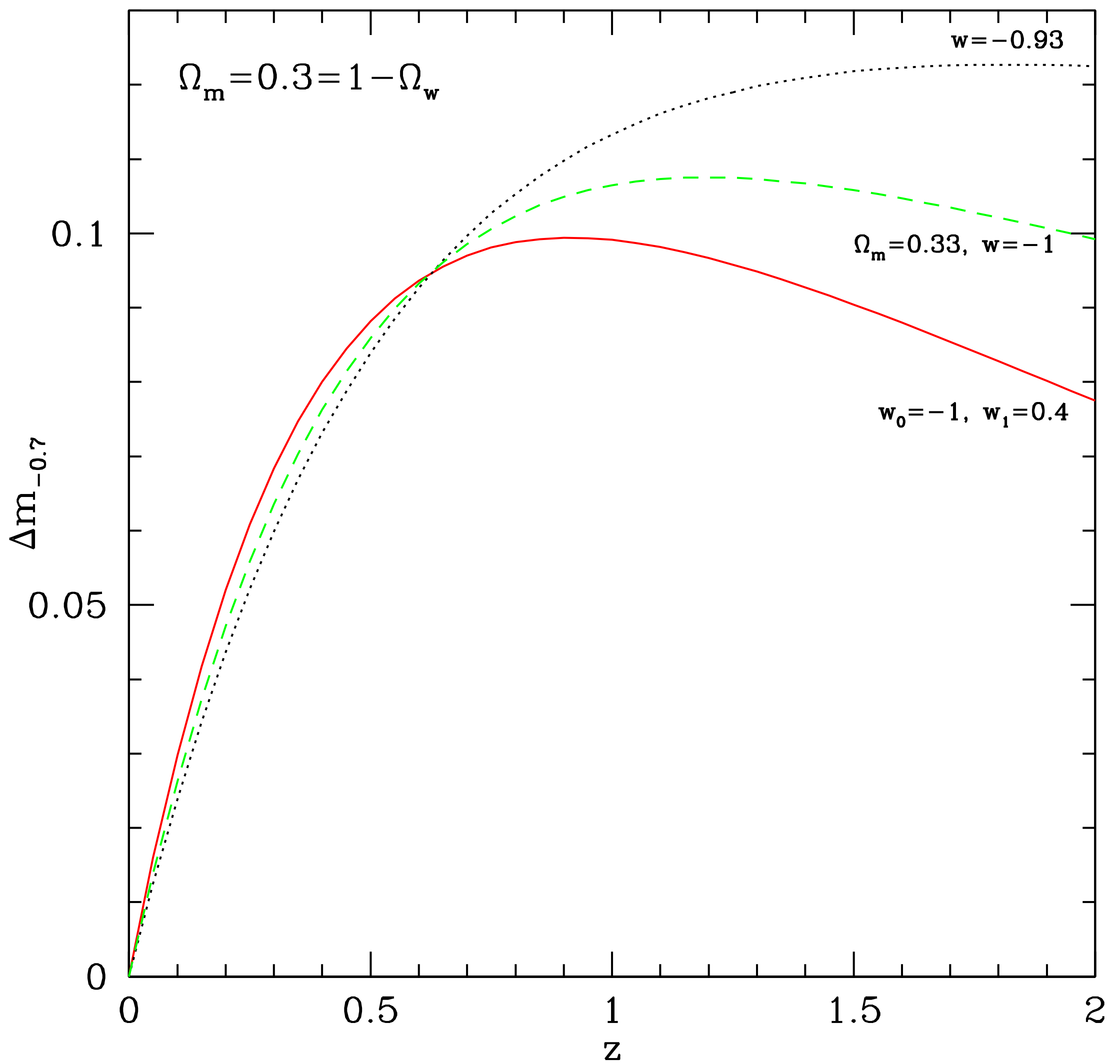

Figure 3: Degeneracies due to the dark energy model, e.g. equation of state value or evolution, and the cosmological model, e.g. value of Ω_m, cannot be resolved at low redshifts. Only at $z \approx 1.7$ do these very different physics models exceed 0.02 mag discrimination.

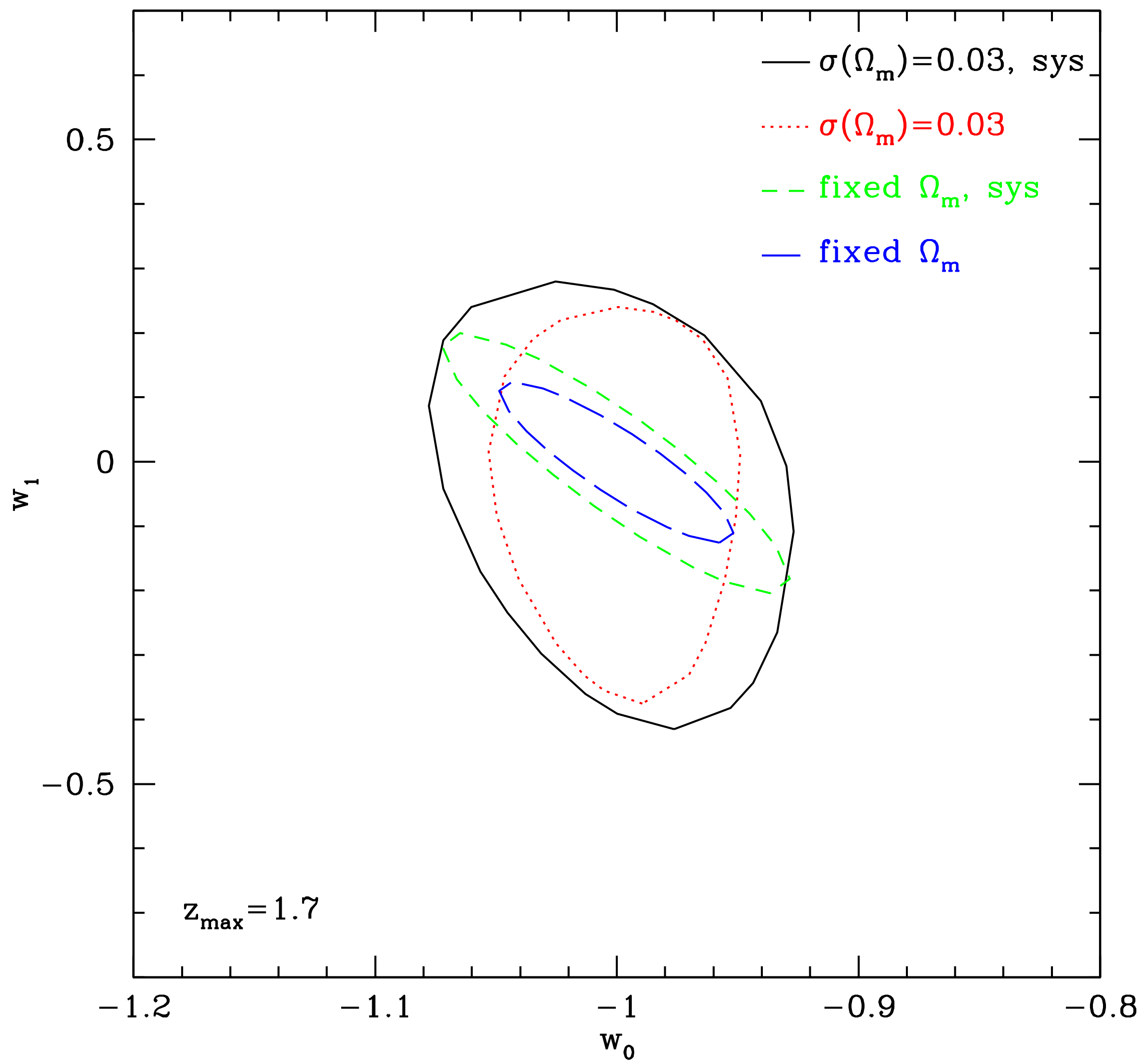

Figure 4: The effect of breaking ideal assumptions for a survey out to $z = 1.7$. Contours are 1σ projected in each variable.

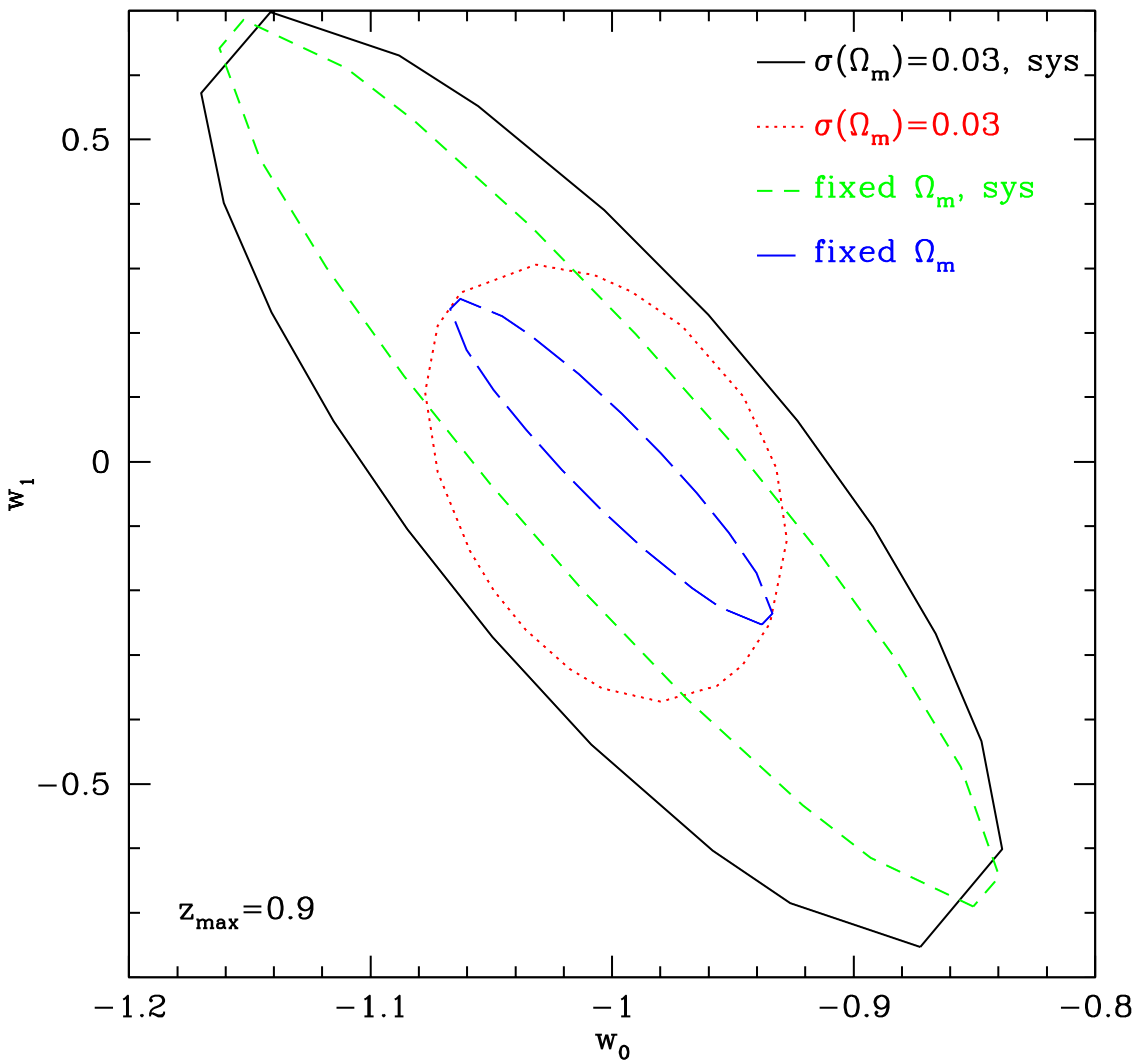

Figure 5: The effect of breaking ideal assumptions for a survey out to $z = 0.9$. Contours are 1σ projected in each variable.

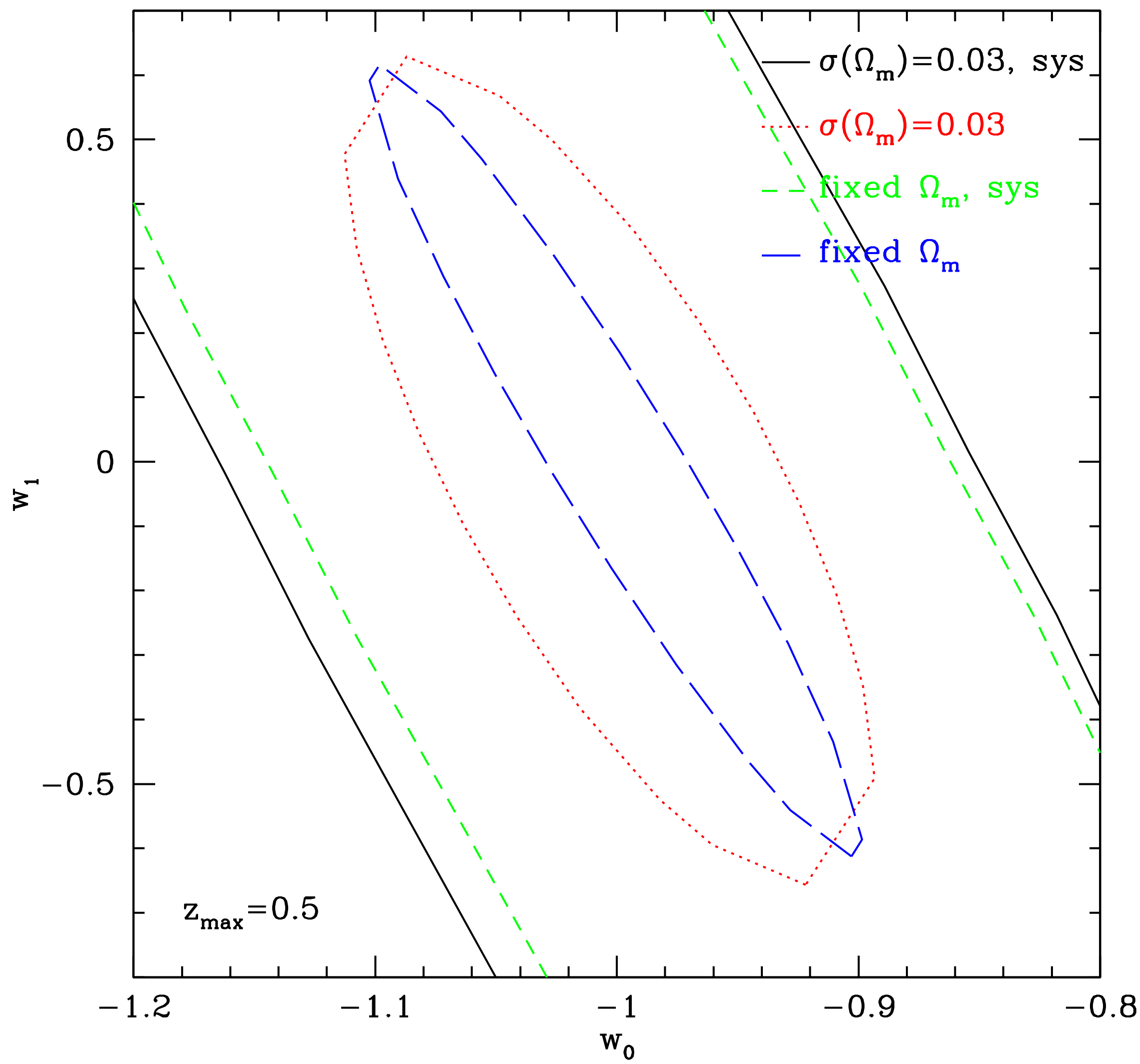

Figure 6: The effect of breaking ideal assumptions for a survey out to $z = 0.5$. Contours are 1σ projected in each variable.

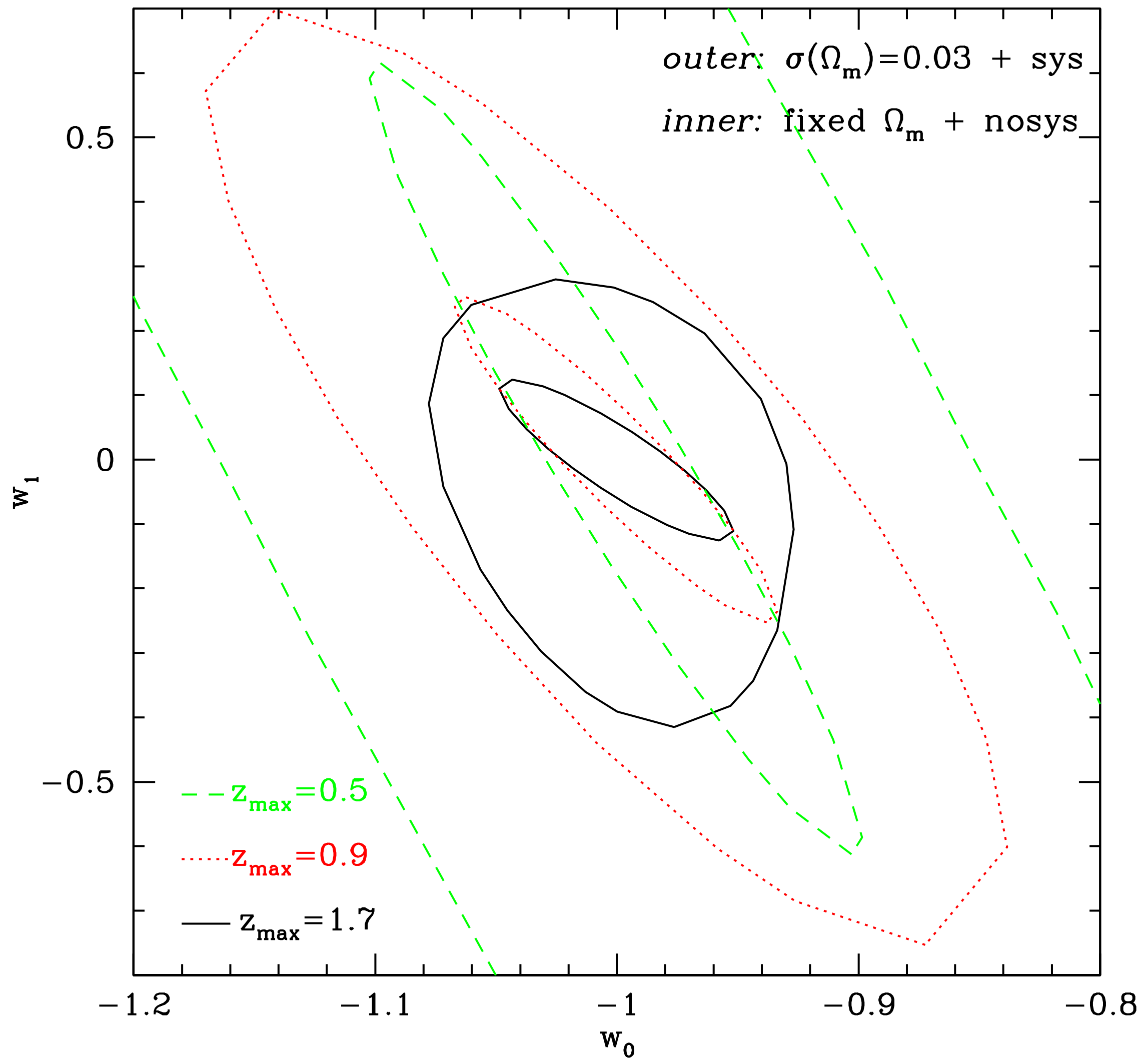

Figure 7: The role of assumptions and survey depth on cosmological parameter determination. Inner contours of each type show ideal assumptions, outer allow realistic breaking. One must be careful in using similar assumptions in comparing experiments: note that an idealized $z = 0.5$ survey appears to do much better than a realistic $z = 0.9$ one. But deeper surveys ameliorate the effect of systematics; a realistic $z = 1.7$ experiment provides comparable limits to an idealized $z = 0.9$ one.