Dynamics Summary

Martin Weinberg

July 09, 1999

- Hamiltonian dynamics
- N-body diagnostics
- What's left…

Newton's Equations and Hamiltonian Dynamics

- *Newton's Laws*: local description of evolution
- *Hamilton's equations*: global description of all possible solutions

Uniquely Hamiltonian viewpoints

- Regular and irregular motion
  - Lyapunov exponents
  - Surface-of-section
- Natural dynamical coordinates
  - Hamilton-Jacobi
  - Actions and Angles
  - Orbit morphology
- Dynamics is motion on a torus
  - Sympletic manifold
  - Better *action-conserving* orbit integration

Application to N-body calculation

- Exploit natural coordinates
- Check whenever possible that dynamical symmetries are observed
- Monitor conserved quantities
  - Energy
  - Actions and bounding integrals of motion

- Equilibria
  - *Collisionless* Boltzmann equation
    - $N \to \infty$
  - *Collisional* Boltzmann equation
    - Dynamical relaxation
    - Fokker-Planck equation
    - Relaxation time
  - Check regime!
  - Jeans' equations
- Diagnostic for equilibrium:

  Virial theorem

  Consider:

$$C = \sum_i \mathbf{p}_i \cdot \mathbf{x}_i$$

Differentiate with respect to time:

$$\begin{aligned}\frac{dC}{dt} &= \sum_i \dot{\mathbf{p}}_i \cdot \mathbf{x}_i + \dot{\mathbf{p}}_i \cdot \dot{\mathbf{x}}_i \\ &= \sum_i \mathbf{F}_i \cdot \mathbf{x}_i + 2T\end{aligned}$$

Now average over time, τ:

$$\begin{aligned}\frac{1}{\tau}\int_0^\tau dt \frac{dC}{dt} &\equiv \langle \frac{dC}{dt} \rangle \\ &= \langle 2T \rangle + \langle \sum_i \mathbf{F}_i \cdot \mathbf{x}_i \rangle \\ &= \frac{1}{\tau}[C(\tau) - C(0)] \to 0\end{aligned}$$

$$\boxed{\langle T \rangle = -\frac{1}{2}\langle \sum_i \mathbf{F}_i \cdot \mathbf{x}_i \rangle}$$

For future study …

- Dynamical computation *besides* simulation
- Evolution by direct solution of Boltzmann eqn.
  - Perturbation theory
  - Fokker-Planck
- Chaos
  - barred galaxies
  - galactic centers
- Multiscale interactions
  - Galaxy components
  - Star formation