Testing Consistency of $\Omega_\Lambda - \Omega_M$ Plots and Flatness

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Dark energy other than a cosmological constant could appear to give an erroneous nonflat result in the $\Omega_\Lambda - \Omega_M$ plane. We show how to recognize this and relate the apparent Ω_k to $w \neq -1$ or $w' \neq 0$.

Current supernova data provide constraints on a set of cosmological parameters, e.g. $\{\Omega_w, \Omega_M, w\}$. One would like to use other types of probes, e.g. CMB and large scale structure in one of two ways: to complement and provide external constraints or to independently compare the cosmological models derived from these different data sets. An example of the latter is a graph in the $\Omega_\Lambda - \Omega_M$ plane where the SN best fit falls off the CMB provided flatness line; in the future one could imagine a high confidence level contour from an increasingly large SN sample, e.g. SNAP, becoming disjoint from the flatness line.

Indeed, original SCP results showed the maximum likelihood at $\Omega_\Lambda \approx 1.3$, $\Omega_M \approx 0.6$, decidedly nonflat. While further data belie this, let us consider the implications of a fit pointing toward a nonflat result. Such an apparent disagreement with CMB results does not necessarily signal a breakdown of the FRW cosmological model. Nonintersection of contours from different probes can occur by other means, even assuming the data are fully corrected for systematic errors. For example, nonadiabatic perturbations (e.g. isocurvature or unaccounted tensor modes) could cause large scale structure constraints not to agree with CMB constraints.

In the case of SN, the $\Omega_\Lambda - \Omega_M$ graph can fail to give a full picture if the assumption $w = -1$ fails. Likewise the $w - \Omega_M$ graph gives a false impression if flatness does not hold or if w is not constant. Let us consider how data could teach us something about the dark energy model, or at least understand how an unexpected result would appear. We initially approach this by arbitrarily assuming that the data are well fit by $\Omega_\Lambda = 1.3$ and $\Omega_M = 0.6$ and asking what the "true" flat model might be. For convenience we assume the true model has constant w. Results are shown in Fig. 1; we can understand them as follows.

At low redshifts, the SN data possess a degeneracy axis along a line of constant $2q_0 = \Omega_M + (1+3w)\Omega_w$. As the sample incorporates higher redshift SN, the axis steepens, effectively reducing the Ω_w contribution by a factor $S(z)$: the line is defined by $\Omega_M + [(1+3w)/S]\,\Omega_w$. In the limit of large z a distance probe is insensitive to the dark energy density or equation of state, cf. the CMB data (for $z \approx 1$ one already has $S \approx 4$). A shift up the axis (to higher Ω_m and Ω_w) defined by a low redshift sample is therefore for a deeper survey a shift to a universe with a more strongly accelerating expansion. That is, a universe with $\Omega_\Lambda = 1.3$, $\Omega_M = 0.6$ is not only less flat but more accelerating than one with $\Omega_\Lambda = 0.7$, $\Omega_M = 0.3$, which also lies along the $S = 4$ degeneracy axis.

Now consider the role of the equation of state w. If one wants to fit a flat $\Omega_w = 0.7$, $\Omega_M = 0.3$ model to data that fit $\Omega_\Lambda = 1.3$, $\Omega_M = 0.6$ then one must "make up for" the reduced dark energy density by emphasizing the negativity of w. Thus, the best flat fit for such data is seen, *strictly from the* $\Omega_\Lambda - \Omega_M$ *plane,* to be $w \approx -1.3$, as shown in Fig. 1. So if we take consistency with the CMB data as a desideratum, then in such a case we are automatically led to the conclusion that the $\Omega_\Lambda - \Omega_M$ plane diagram is not the whole story. In general a true, flat, $w < -1$ model will show up as a drift above the flatness line in that plane, and a flat, $w > -1$ model will lie below the flatness line.

Therefore one should always also present the $w - \Omega_M$ plane simultaneously to show whether the SN data are truly testing consistency with the CMB results. If the likelihood peak in this plane lies at -1, then it is a true cosmological model consistency test (forgetting for the moment about w' etc.). But if the peak is off -1 then there is an indication that the cosmological constant is not the correct dark energy model. Moreover, if the peak lies off -1 in the appropriate direction and by the appropriate amount then this also supplies evidence for consistency with flatness for a dark energy characterized by that w.

From the original SCP $w - \Omega_M$ data one in fact seemed to obtain a best fit near $w = -1.3$ in this plane, consistent with the CMB flatness data. We emphasize that this is merely an illustration and one should not conclude that indeed $w < -1$ is preferred. The main point is that by simple simultaneous examination of the two graphs one has a straightforward test of cosmological consistency and a further insight into the dark energy model. A three dimensional plot $\Omega_w - \Omega_M - w$ shows this as well but is less easy to interpret visually.

What about an improved data set? Already from Fig. 1 one can deduce that either reduced errors within the current survey depth or, more powerfully, extension of the redshift range will enable a true consistency test for flatness and $w \neq -1$. A search garnering several tens of supernovae at moderate redshifts $z < 1$ could lead to a drastic improvement in the $w - \Omega_M$ plane, especially on the more sensitive $w < -1$ side.

Finally, let us address two flies in the ointment. The presence of a variation in the equation of state, w', does not upset the present situation very much. To mock up a constant $w \approx -1.3$ one would require either an even more negative w_0 and a positive w' or v.v., say a $w_0 \approx -1$ with $w' \approx -1.2$; both of these seem extreme. The general trading rules to mock up another variable are roughly $dw' = -4\,dw_0$ and $d\Omega_k = -3\,dw$ (for $z \approx 1$ and a model near a flat $\Omega_M = 0.3$, $w = -1$ one). Secondly, the contours from the CMB interpreted as flatness constraints do depend to some extent on w as well, so if the true model is not a cosmological constant then these too should be considered in the $w - \Omega_M$ plane, or adjusted for the $\Omega_\Lambda - \Omega_M$ plane. However, the CMB results are not very sensitive to w (roughly $d\Omega_k = 0.1\,dw$) so this is likely to be a small effect.

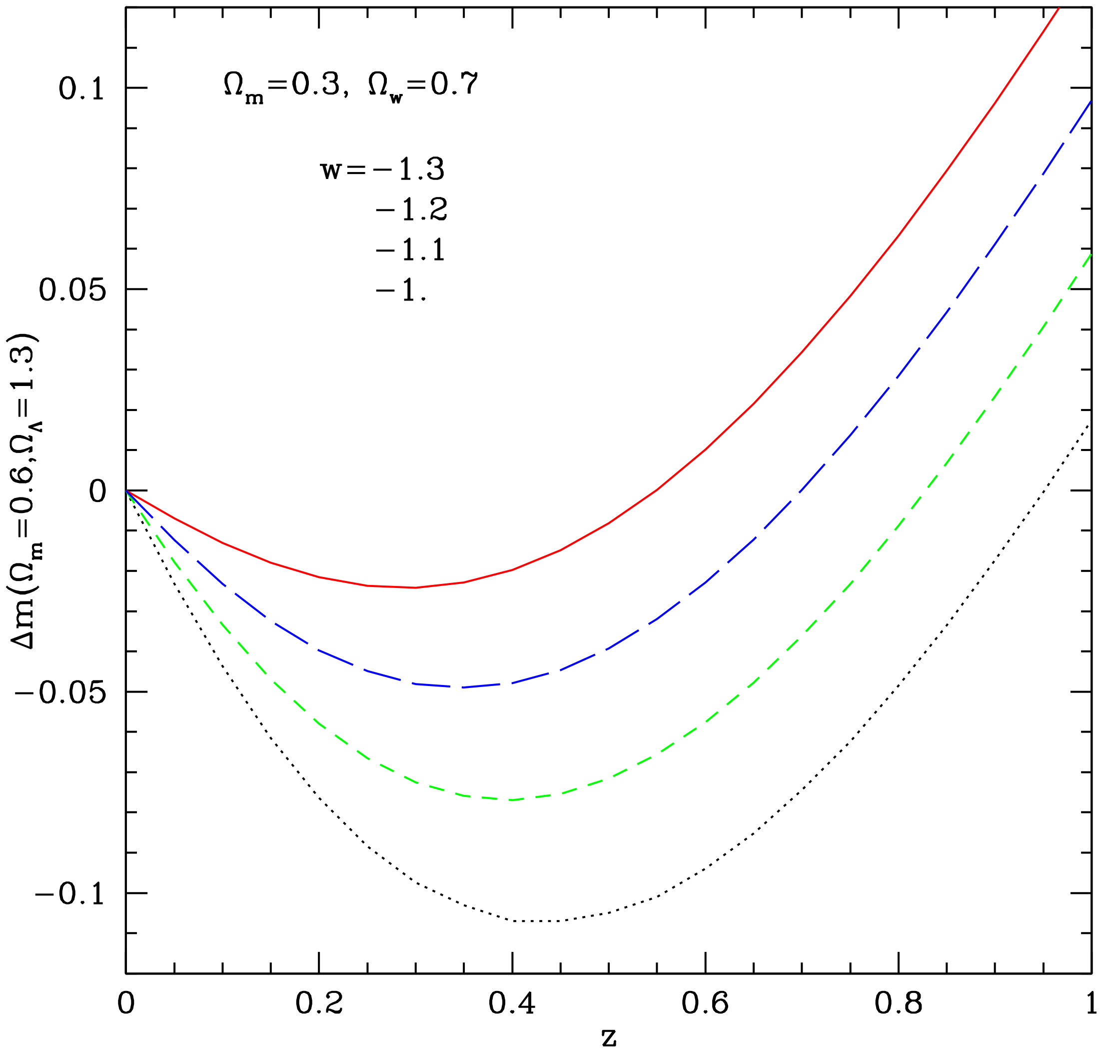

Figure 1: Magnitude offset of flat models relative to a particular nonflat best fit. A fit $\Omega_M = 0.6$, $\Omega_\Lambda = 1.3$ obtained over a limited redshift range could actually indicate a flat cosmology with $w < -1$. Note that if the $w = -1$ flat model (dotted line) lay near the 1σ contour in the $\Omega_\Lambda - \Omega_M$ plane (as for the original 42 SCP supernovae), then the $w < -1$ models also provide excellent fits. From this plot we can already deduce that reduced errors within the current survey depth or an increased redshift range will enable a true consistency test for flatness and $w \neq -1$.