Cosmological parameters as a function of z

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ABSTRACT

The redshift depth of a supernova search affects the precision of the cosmological parameter measurements (Ω_M, Ω_Λ, Ω_k, w_0, and w_1 where $w = w_0 + zw_1$). If the errors involved were purely statistical, there would be no limit to how well the parameters could be measured. For any range of redshift, a sufficiently large number of supernovae could be used to beat down the statistical errors to any desired precision. In reality, there will be systematic errors that limit cosmological parameter determination. In this note, we use simple models for the systematic errors to explore their effect on parameter measurements as a function of limiting redshift.

1. Error Models and Parameter Determinations

In this Section, we consider measurements of the parameters Ω_M, Ω_Λ, and Ω_k where the dark energy is assumed to be the cosmological constant. An additional fit parameter corresponding to the absolute magnitude of the Type Ia has been marginalized in our parameter determination. The errors considered here are systematic limited, with the assumption that enough supernovae can be studied to make statistical errors negligible.

1.1. Errors uncorrelated in redshift

There are two models of systematic error. The first are 100% correlated errors within each redshift bin, 0% correlation outside. In other words, in each redshift bin there is an irreducible systematic error drawn from a Gaussian distribution independent of other redshift bins. K-correction error is a quasi-example of this kind of systematic. The same K-corrections will be applied to supernova magnitudes at a single redshift; their errors will be uncorrelated for very different redshifts.

Different systematic error sizes and evolution in redshift as well as differing redshift bin sizes have been examined.

1. The redshift bin is 0.01 in size and there is an irreducible 0.02 magnitude error in each bin, uncorrelated with the others. Referred to as "Case 1" (Figure 1).
2. There are 0.01 redshift bins with an error model of $0.02 * z/z_{max}$. The magnitude uncertainty always peaks to the same value at the limiting redshift (Figure 2).
3. There are 0.01 redshift bins with an error model of $0.02 * z/1.7$. The error for a given redshift is independent of z_{max} (Figure 3).

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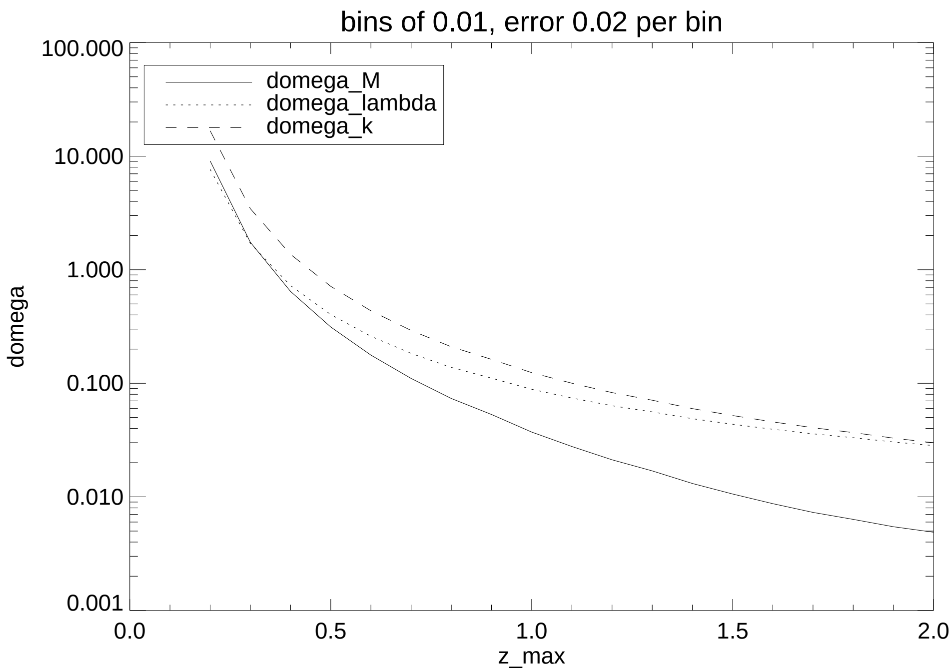

Fig. 1.— The redshift bin is 0.01 in size and there is an irreducible 0.02 magnitude error in each bin, uncorrelated with the others.

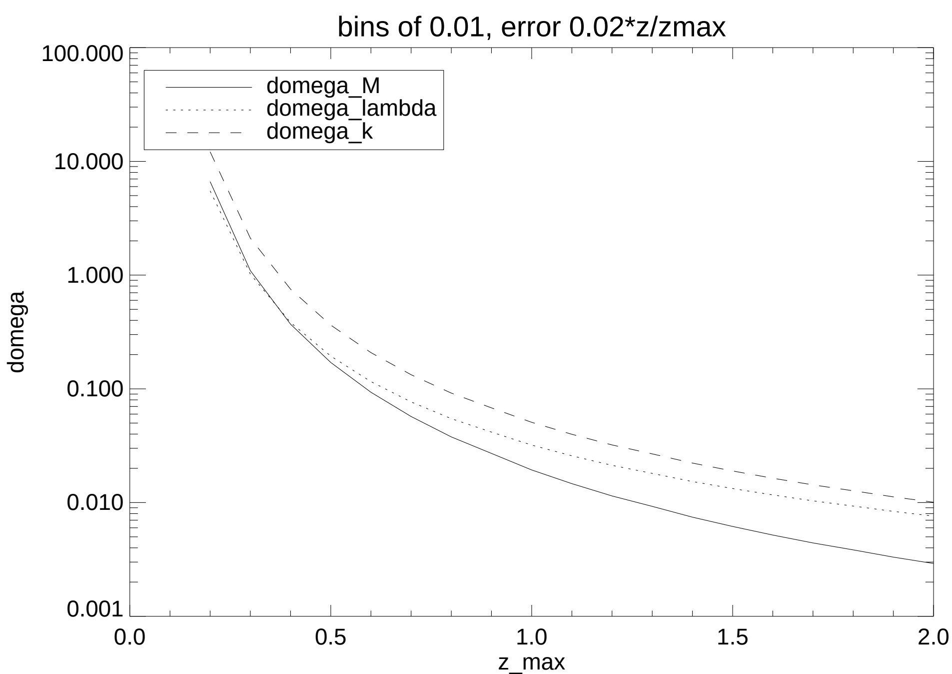

Fig. 2.— There are 0.01 redshift bins with an error model of $0.02 * z/z_{max}$.

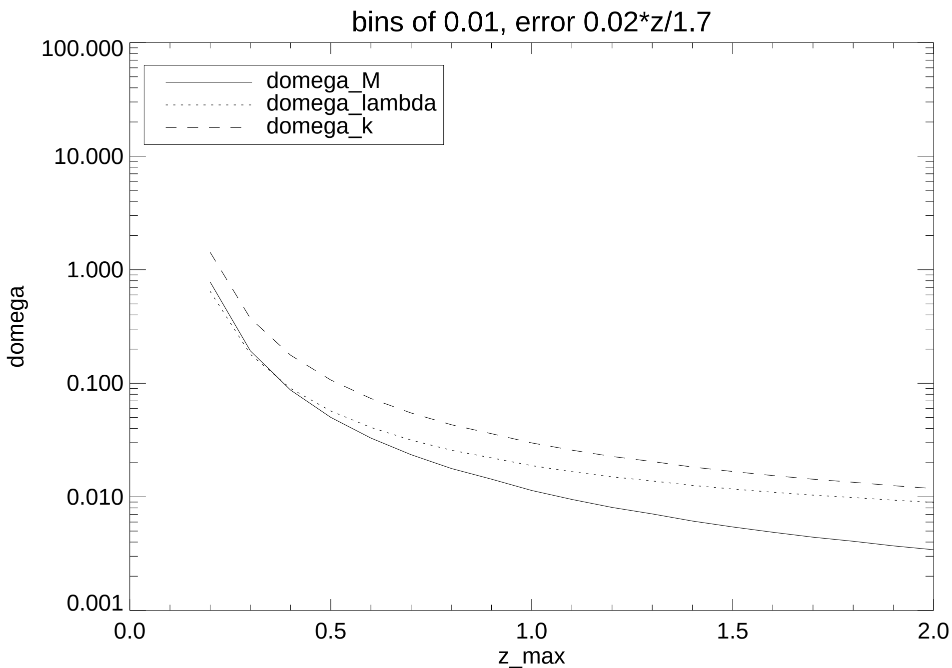

Fig. 3.— There are 0.01 redshift bins with an error model of $0.02 * z/1.7$.

The redshift resolution is fine enough so that changing the fundamental bin width is equivalent to increasing the number of observed supernova. Figure 4 plots parameter errors from the first (constant) error model with 0.03 redshift bins. Overplotted are the errors from Case 1 multiplied by $\sqrt{3}$. The overlapping lines show that 0.02 magnitude error in 0.03 redshift bins is identical to $\sqrt{3} * 0.02$ magnitude errors in 0.01 redshift bins.

It should be noted that the improvement in cosmological parameter determination with increasing maximum redshift is not merely because we are using more supernovae. In Figure 5 we examine $\sqrt{N_{SN}}\, d\Omega$ as a function of redshift and find that it continues to decrease out to $z = 2$. A wider baseline in redshift improves parameter determination beyond what is expected from simply the increased number of supernovae used in the analysis.

1.2. Errors correlated in redshift

The second class of model is a completely correlated systematic magnitude shift. The errors in each redshift bin are not drawn from a Gaussian distribution but are modeled as a linear function of redshift. This model is motivated by systematic errors that may be expected to increase monotonically with redshift (e.g. supernova evolution, Malmquist bias, gray dust). We determine the best cosmological fit given a linear magnitude deviation from an input $\Omega_M = 0.72$, $\Omega_\Lambda = 0.28$ model for the following cases.

1. The deviation is $0.02 * z/z_{max}$. The largest systematic error is scaled to the limiting redshift. (Figure 6.)

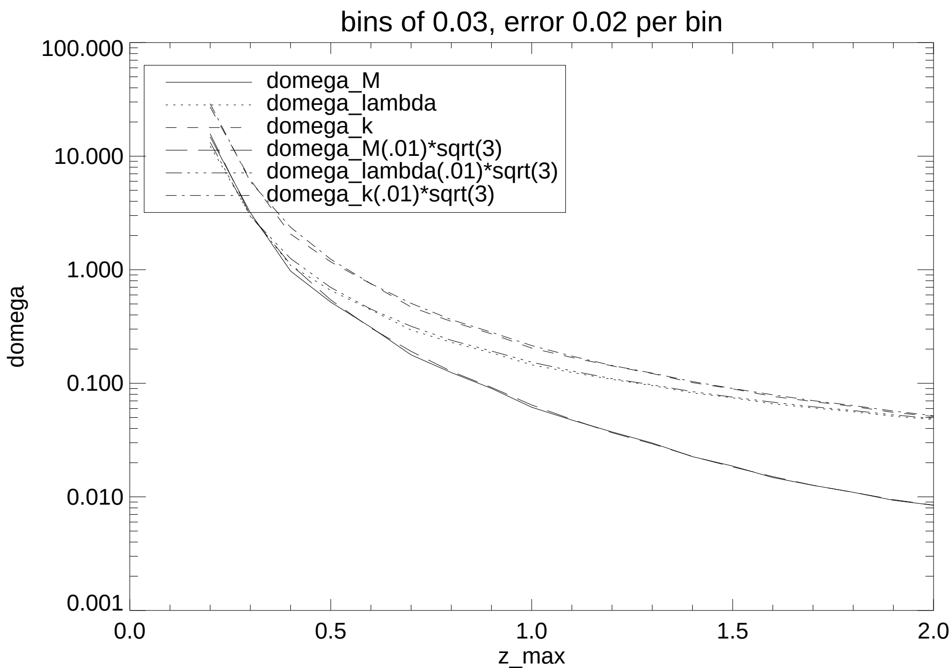

Fig. 4.— The redshift bin is 0.03 in size and there is an irreducable 0.02 magnitude error in each bin, uncorrelated with the others. Overplotted are $\sqrt{3}\times$ Case 1 errors.

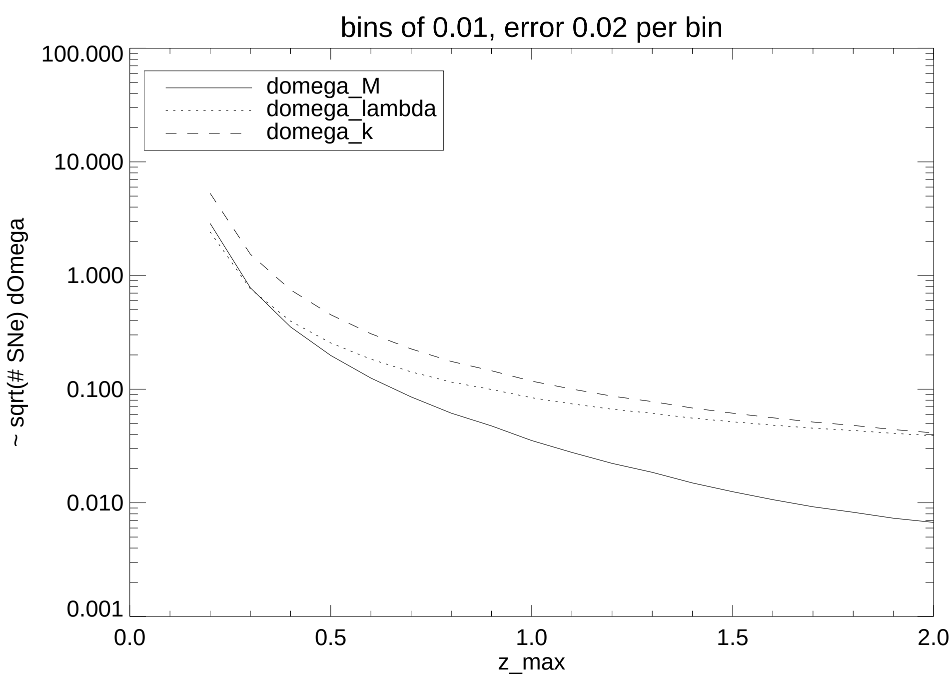

Fig. 5.— $\sqrt{N_{SN}}\, d\Omega$ for Case 1. Note the continuing decrease over the redshift range considered.

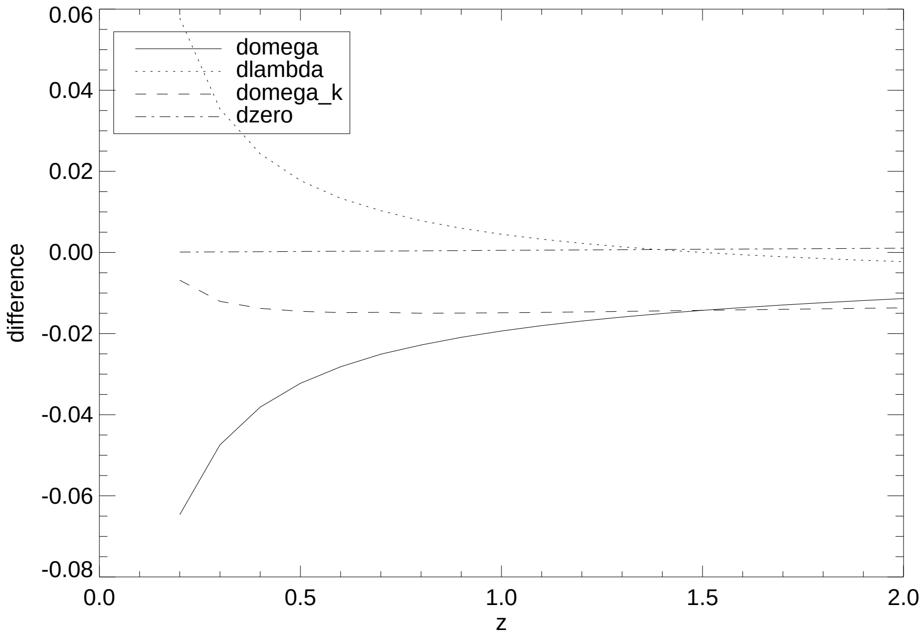

Fig. 6.— There is a systematic magnitude deviation given as $0.02 * z/z_{max}$. The dzero line is $\mathcal{M}$, the normalized supernova absolute magnitude which is simultaneously fit.

2. The deviation is $0.02 * z/1.7$. That is, the systematic errors are independent of limiting redshift. (Figure 7.)

2. Measuring w_0 and w_1

Detailed measurements of the parameters that describe the dark energy require priors. In the following analysis, we assume that Ω_M and Ω_k (and hence Ω_w) are known from independent experiments. The supernova absolute magnitude is still marginalized. (A prior absolute magnitude improves w measurements by a factor of two.)

Figures 8, 9, and 10 correspond to the three scenarios described in §1.1 where the errors are Gaussian and uncorrelated between each redshift bin. Determinations of w_0 are an order of magnitude better than w_1. The advantage of moving to higher redshifts is less pronounced than for the measurements of Ω_M and Ω_Λ and can be seen in the flattening at high redshifts of the curves in Figure 11.

The effect of systematic magnitude biases in redshift as in §1.2 for the measurement of $w = w_0 + w_1 z$ is shown in Figures 13, 12. With increasing z_{max} neither uncertainties in w_0 nor w_1 grow strongly. Determinations are at the level of 0.02 and 0.15 respectively for a background model close to a cosmological constant. Errors are less for more positive equations of state since the dark energy retains its influence to higher redshifts. Similarly, increasing Ω_w of the model would also reduce the uncertainties caused by the systematic errors.

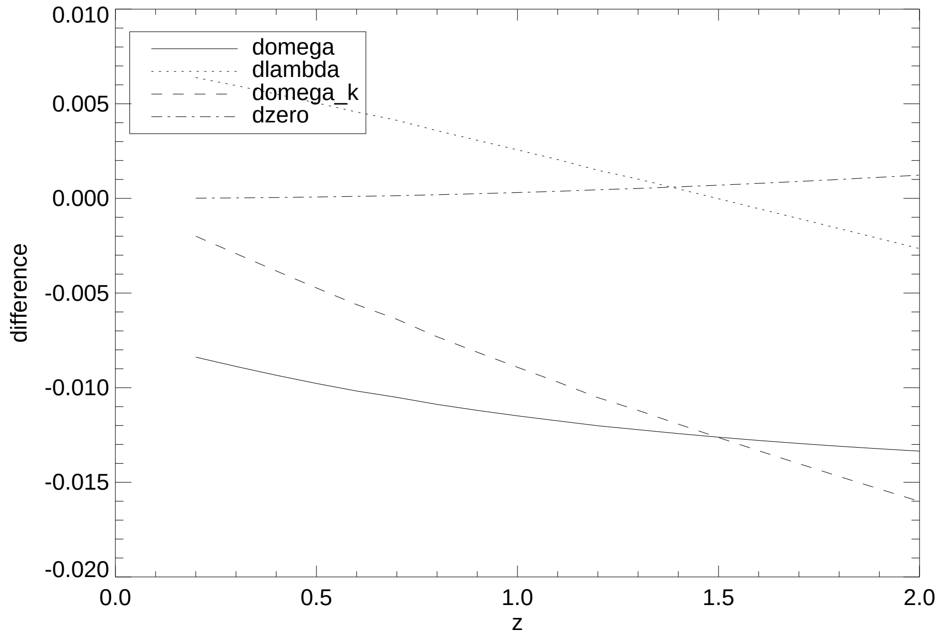

Fig. 7.— There is a systematic magnitude deviation given as $0.02 * z/1.7$.

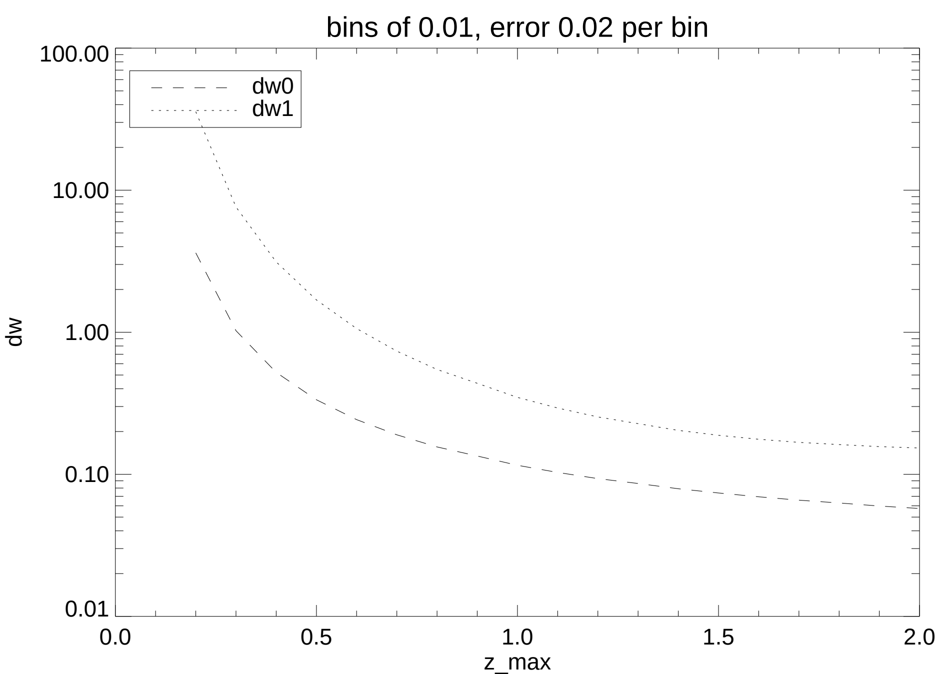

Fig. 8.— The redshift bin is 0.01 in size and there is an irreducable 0.02 magnitude error in each bin, uncorrelated with the others.

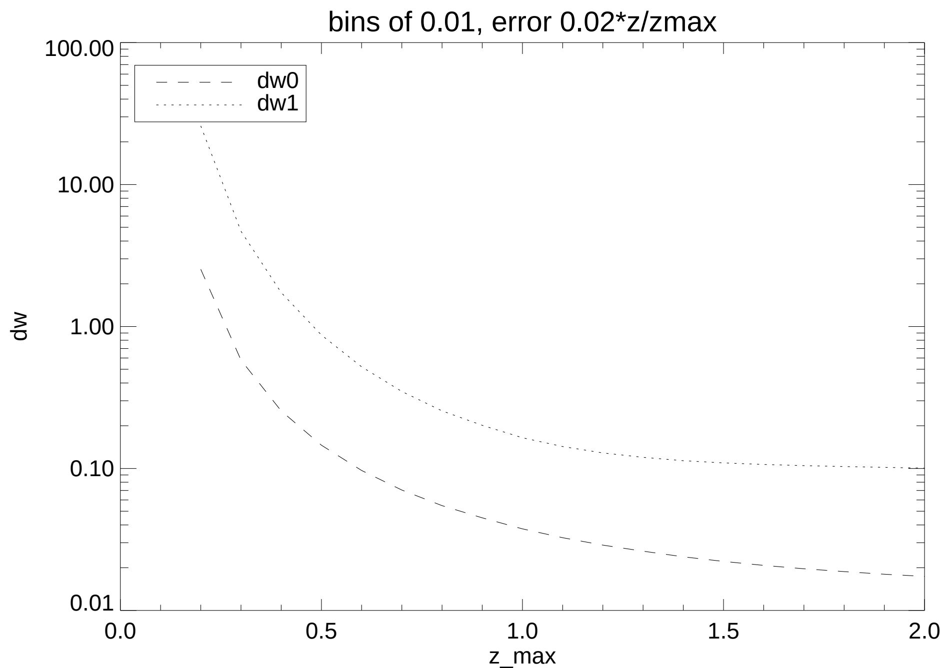

Fig. 9.— There are 0.01 redshift bins with an error model of $0.02 * z/z_{max}$.

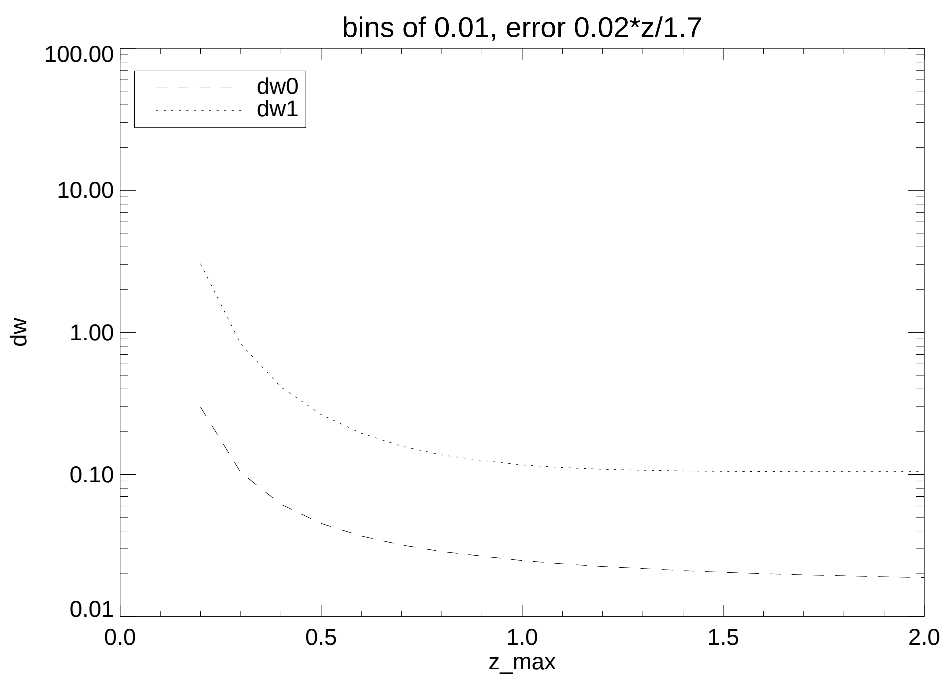

Fig. 10.— There are 0.01 redshift bins with an error model of $0.02 * z/1.7$.

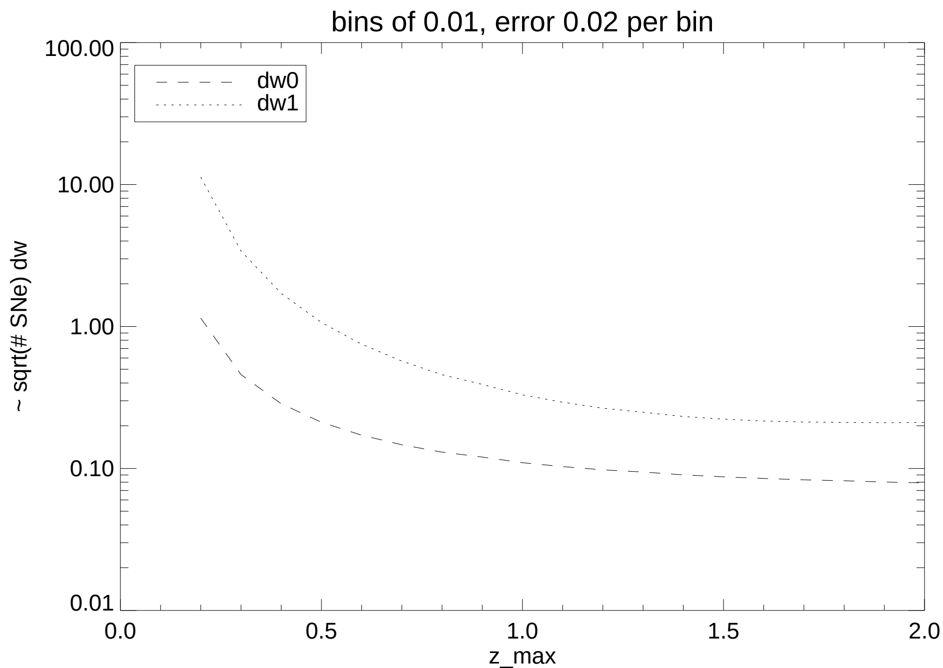

Fig. 11.— $\sqrt{N_{SN}}\, dw$ for Case 1. As compared to Figure 5 the power of an extended range to resolve w_0 and w_1 flattens at high redshift (though it is comparable to that for Ω_Λ).

3. Conclusions

We first note the exceptional case of a systematic magnitude deviation given as $0.02 * z/1.7$. Errors in the cosmological parameters here increase since non-random errors steadily increase for greater z_{max}. The degree of freedom in $\mathcal{M}$ steadily absorbs the mean magnitude shift while the cosmological parameters diverge from their input values.

Parameter determination improves with larger z_{max} for all other systematic error scenarios, due to the better leverage, increased "statistics", and smaller errors for a fixed redshift. Within the range of models explored, a $z_{max} = 1.7$ survey can in principle reach $d\Omega_X = 0.01 - 0.03$. Ground-based searches that target $z_{max} = 0.5 - 1.0$ will do a factor of 5-10 worse. The same advantage exists but is less pronounced in the measurements of w_0 and w_1. A $z_{max} = 1.7$ survey can give $dw_0 = 0.01 - 0.03$ and $dw_1 = 0.1 - 0.2$ whereas ground-based searches from $z_{max} = 0.5 - 1.0$ will do a factor of a few worse.

The particular nature of the systematic errors has not been determined. However, their magnitudes will be constrained by the measurement of observable trackers of systematics such as light-curve rise time, plateau levels, and spectral features.

The number of supernovae required to achieve the statistical limit is large. To achieve a 0.02 magnitude bin uncertainty assuming ~ 0.1 statistical uncertainty in individual corrected Type Ia peak magnitudes requires 25 supernovae. SNAP will have these statistics for 0.01 redshift bins from $0.8 \leq z \leq 1.2$ in our seventeen month baseline mission. This is independent of other systematics, such as gravitational lensing, that will increase the observed magnitude dispersion.

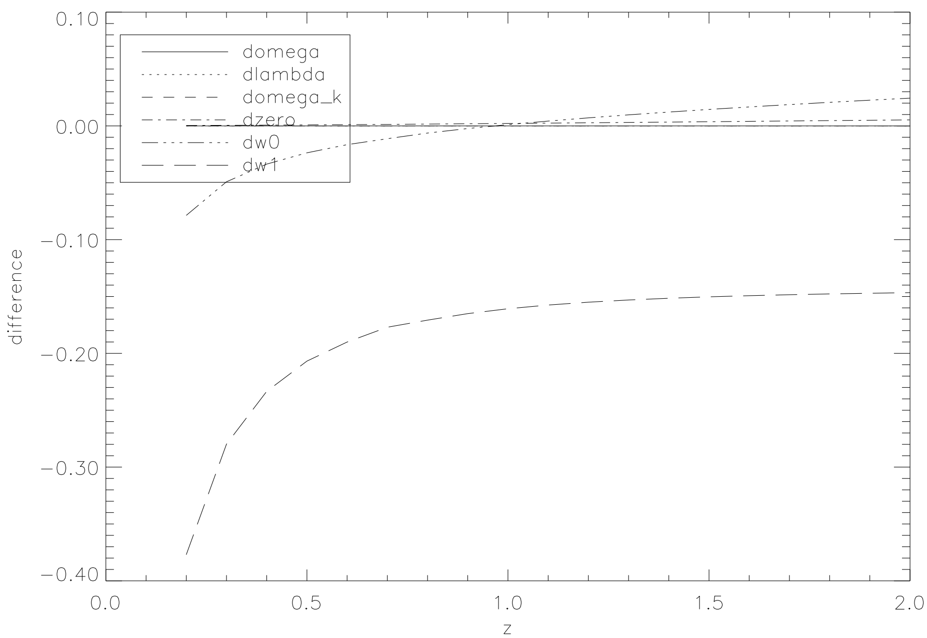

Fig. 12.— The systematic magnitude deviation is given by $0.02 * z/z_{max}$. $\Omega_m = 0.28$ and $\Omega_w = 0.72$ are held fixed but $\mathcal{M}$ can vary, shown as dzero.

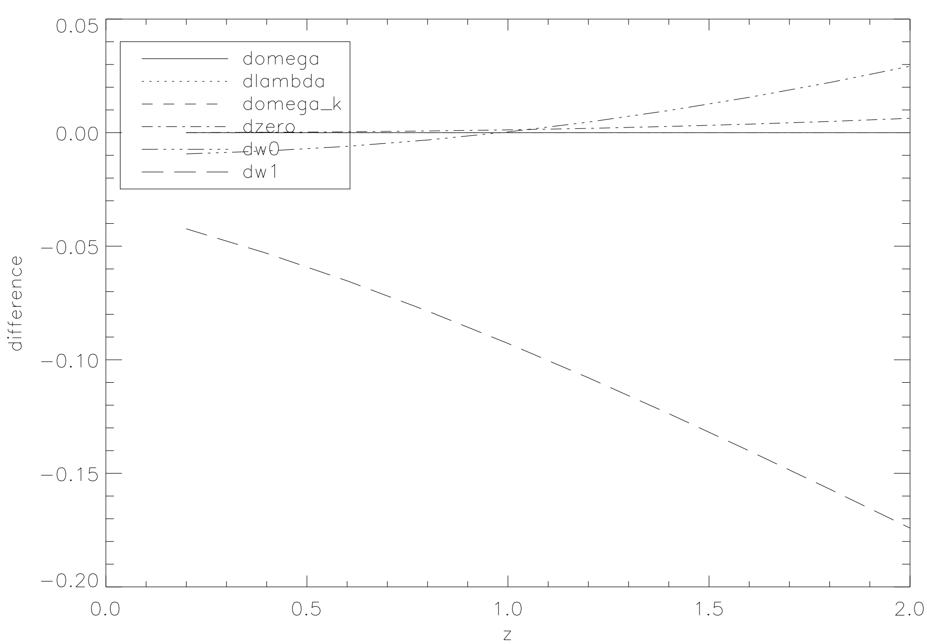

Fig. 13.— The systematic magnitude deviation is given by $0.02 * z/1.7$. $\Omega_m = 0.28$ and $\Omega_w = 0.72$ are held fixed but $\mathcal{M}$ can vary, shown as dzero.