From Data to Theory (and back)

To compare observations and theory we need a statistical measure of goodness of fit.

We need to compare the theory value, e.g. for distance-redshift,

$$d_{lum} = (1+z) \int_0^z dz' \,/\, H(z'; \Omega_m, w(z'))$$

to the data D_{lum}^i. For example χ^2 or likelihood

$$\chi^2 = \sum_{i,j} [D_{lum}^i - d_{lum}(z_i)] \, COV^{-1}(i,j) \, [D_{lum}^j - d_{lum}(z_j)]^t$$

$$L = \exp(-\chi^2/2)$$ [Gaussian near max likelihood]

We need 1) theory or robust parametrization w(z), 2) efficient method for estimating parameter errors given data characteristics.

Fisher Matrix

Fisher matrix gives lower limit for Gaussian likelihoods, quick and easy.

See: Tegmark et al. astro-ph/9805117
Dodelson, "Modern Cosmology"

$$F_{ij} = d^2 (- \ln L) / dp_i\, dp_j = \sum_O (dO/dp_i)\, COV^{-1}\, (dO/dp_j)$$

$$\sigma(p_i) \geq 1/(F_{ii})^{1/2}$$

Example: $O=d_{lum}(z=0.1,0.2,\ldots 1)$, $p=(\Omega_m,w)$, $COV=(\delta d/d)d\ \delta_{ij}$
$F_{\Omega w}=\sum_k (dO_k/d\Omega)(dO_k/dw)\sigma_k^{-2}$

$$F = \begin{pmatrix} F_{\Omega\Omega} & F_{\Omega w} \\ F_{w\Omega} & F_{ww} \end{pmatrix} \qquad C = F^{-1} = \begin{pmatrix} \sigma^2(\Omega) & COV(\Omega,w) \\ COV(\Omega,w) & \sigma^2(w) \end{pmatrix}$$

Also called information matrix. Add independent data sets, or priors, by adding matrices.

e.g. Gaussian prior on $\Omega_m = 0.28 \pm 0.03$ via $\chi^2 = (\Omega_m - 0.28)^2/0.03^2$

Survival of the Fittest

Fisher estimates give a N-dimension ellipsoid.

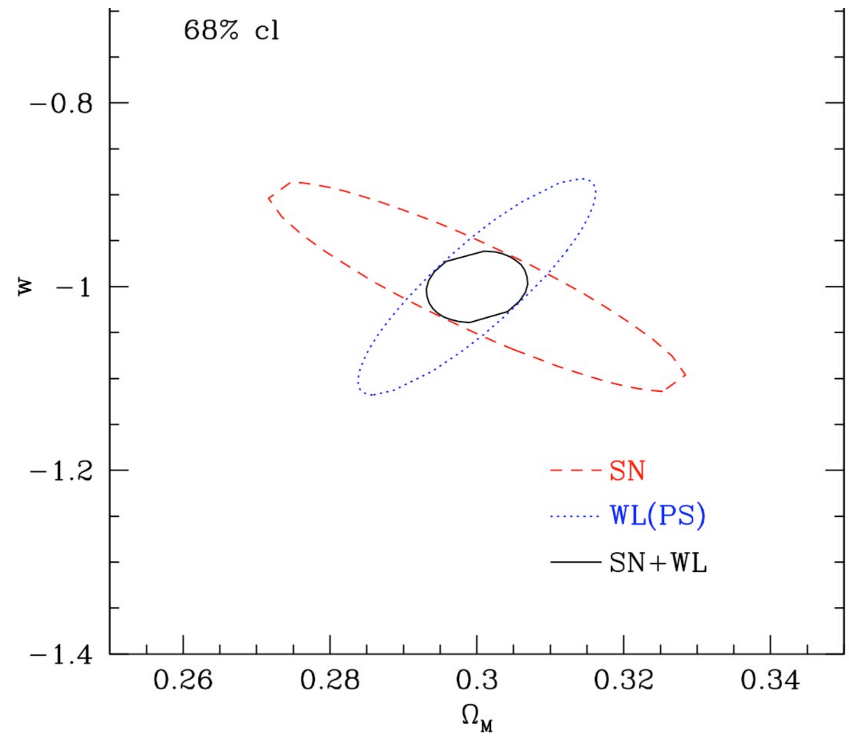
Marginalize **(integrate over the probability distribution) over parameters not of immediate interest by crossing out their row/column in F^{-1}.**

Fix **a parameter by crossing out row/column in F.**

1σ (68.3% probability enclosed) joint contours have $d\chi^2$=2.30 in 2-D (not $d\chi^2$=1). Read off 1σ errors by projecting to axis and dividing by 1.52=$\sqrt{2.30}$.

Orientation of ellipse shows degree of covariance (degeneracy).

Different types of observations can have different degeneracies (complementarity) and combine to give tight constraints.



Model Independence

We could check each theoretical model one by one against the data -- but there are 10^x of them, each with their own parameters. We'd also like to predict / design results of different experiments.

Want model independent approach. Remember

$$H(z)=[\Omega_m(1+z)^3 + \Omega_w \exp\{3\int_0^z d\ln(1+z)\,[1+w(z)]\}\,]^{1/2}$$

Parametrize w(z). Keep close to the physics: both energy density and pressure enter the dynamics; directly related to kinetic/potential energy of scalar field.

Model Independence

Simplest parametrization, with physical dynamics,

$$w(a)=w_0+w_a(1-a)$$

Recall $a=(1+z)^{-1}$.

Virtues:

- Model independent
- Excellent approximation to exact field equation solutions
- Robust against bias
- Well behaved at high z

Problems: Cannot handle rapid transitions or oscillations.

N.B.: constant w lacks important physics;
$w(z)=w_0+w_1z$ is Taylor expansion about low z only - pathological at high z.

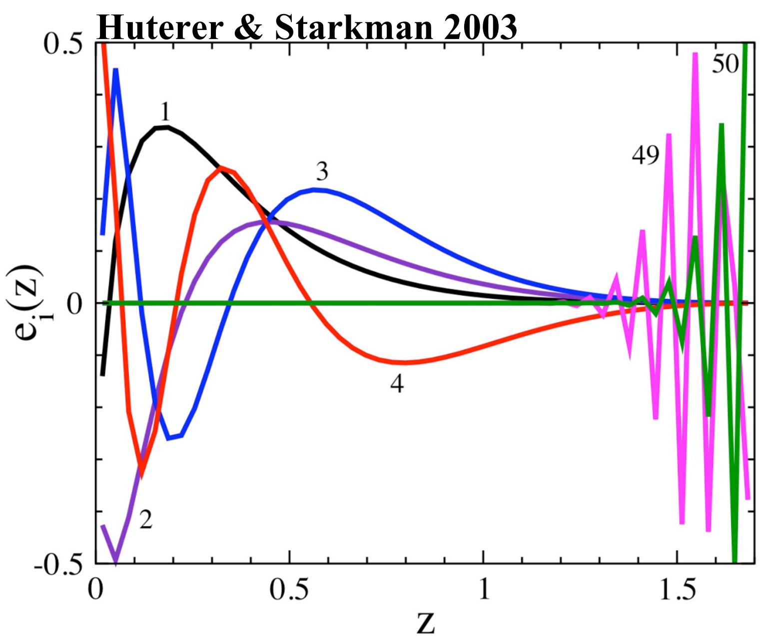
Eigenmodes

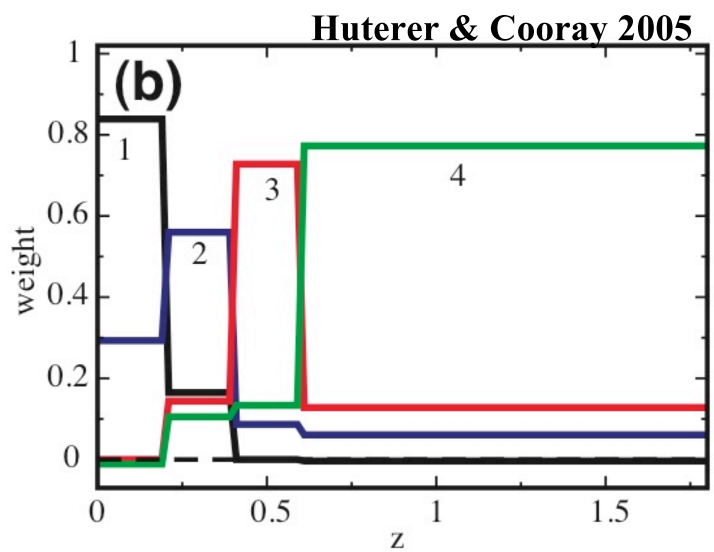
w_0, w_a makes for easiest, robust comparison. But sometimes want nonparametric form.

Eigenmodes of w(z) give independent principal components (but depend on model, experiment, and probe).

Start with parameters of w_i in z bins. Diagonalize Fisher matrix $F=E^TDE$: D is diagonal, rows of E give eigenvectors.

$w(z) = \sum b_i e_i(z)$

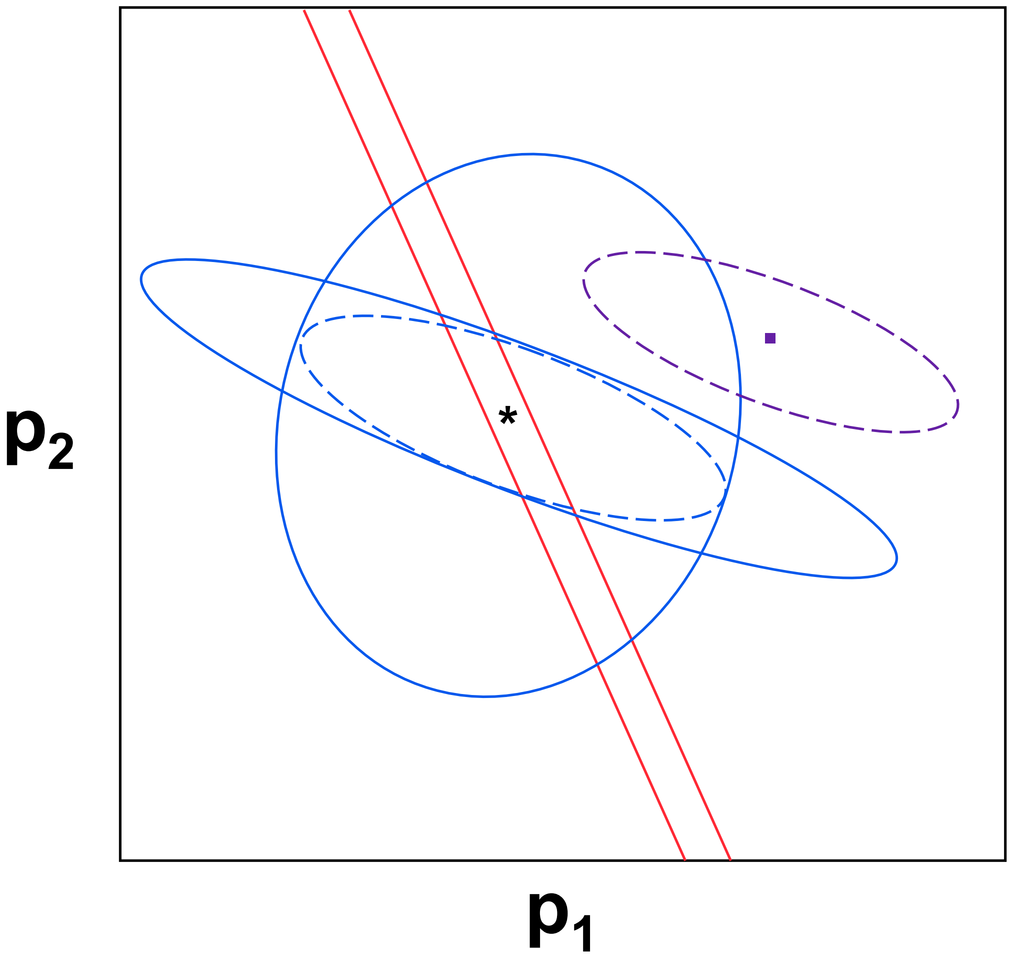




Localized eigenmodes $L=E^TD^{1/2}E$

Design an Experiment

Precision in measurement is not enough - one must beware degeneracies and systematics.



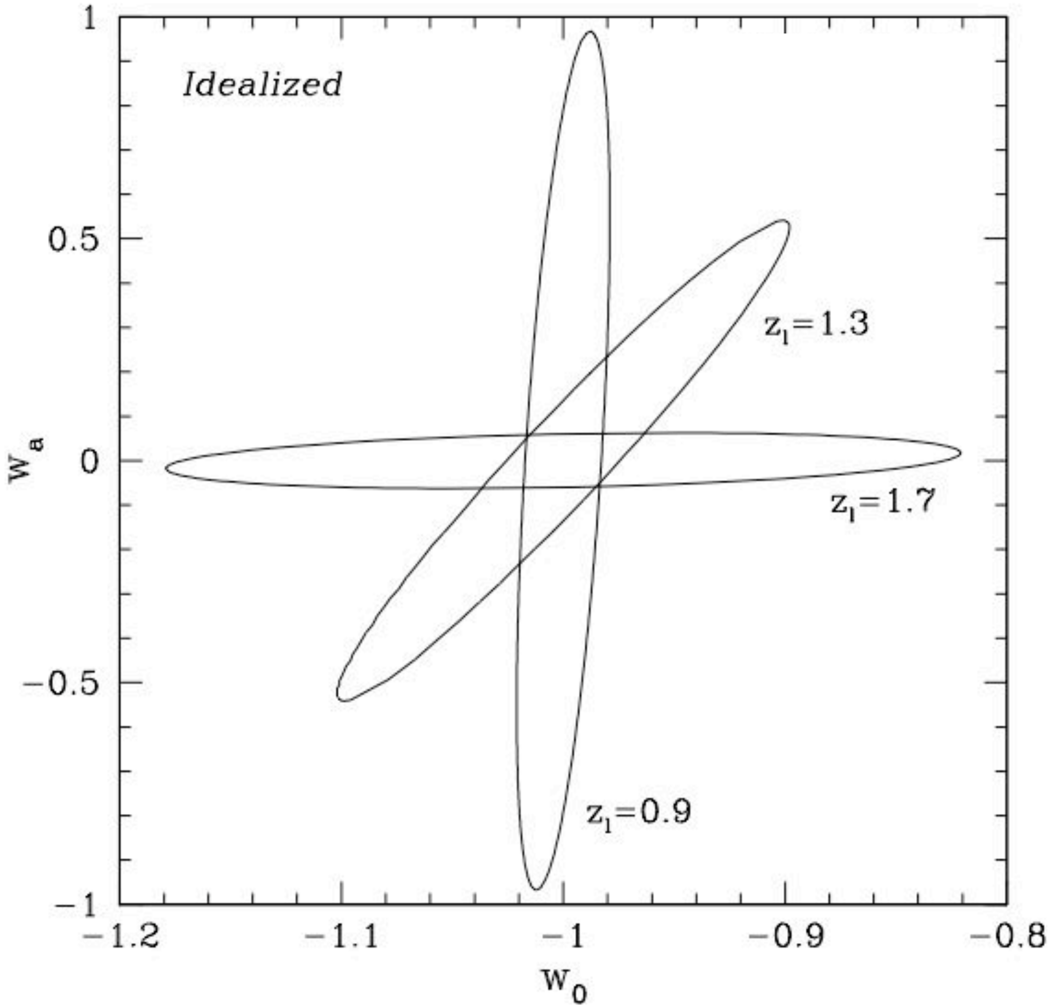
Degeneracy:
e.g. Aw_0+Bw_a=const

Degeneracy: hypersurface, e.g. covariance with Ω_m or Systematic: floor to precision, e.g. calibration

Systematic: offset error in data or model, e.g. evolution

Mapping History

Data over a range of redshifts can be effective at breaking degeneracies. Plus one gets leverage from a long baseline in expansion history.



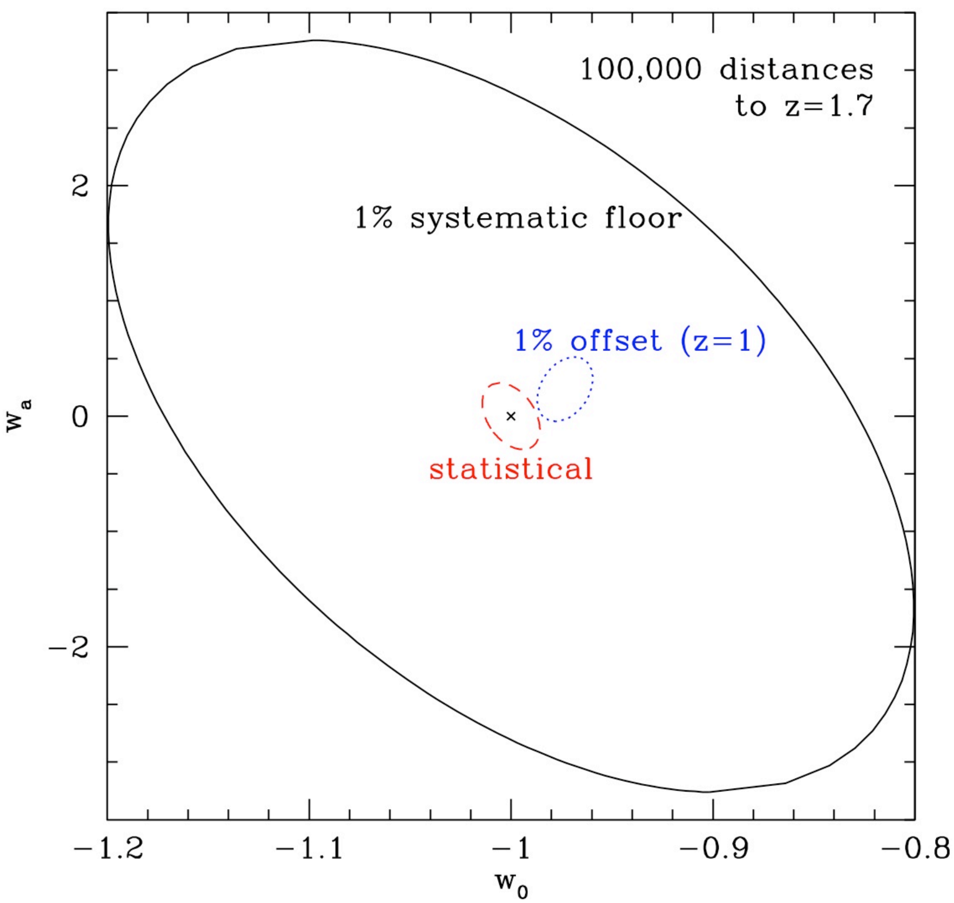
Controlling Systematics

Controlling systematics is the name of the game. Finding more objects is not.

Must understand the sources, instruments, and the theory interpretation.

Forthcoming experiments may deliver 100,000s of objects. But uncertainties do not reduce by 1/√N.

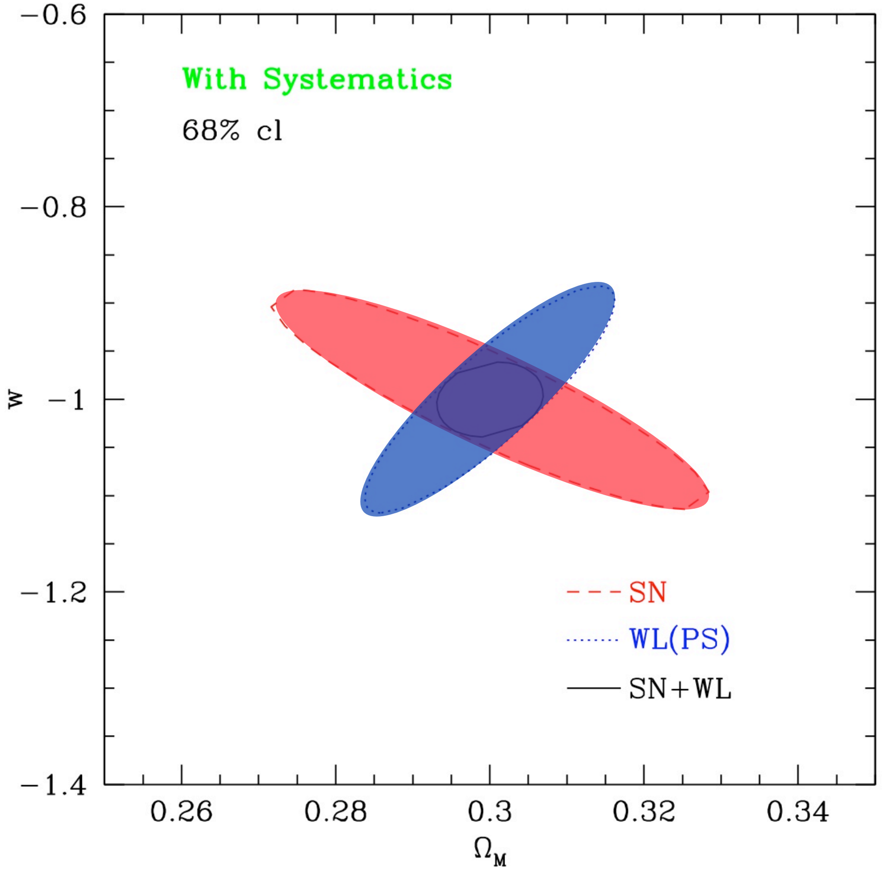
Must choose cleanest probe, mature method, with multiple crosschecks.



Complementarity

Complementarity of techniques (e.g. SN,WL,CMB,…)

- improves precision
- breaks degeneracies
- immunizes against systematics



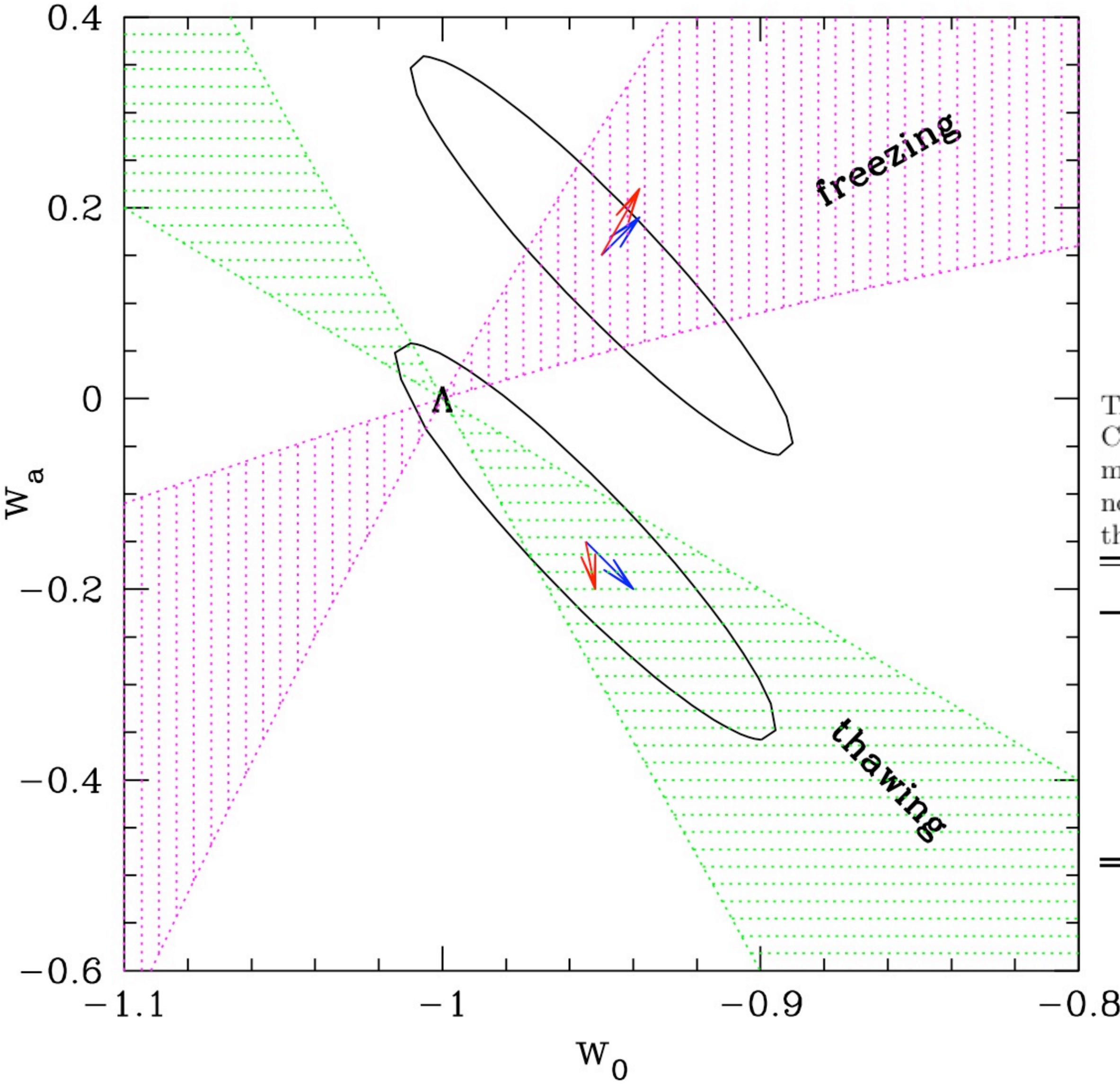
Design an Experiment

How to design an experiment to explore dark energy?

- Choose clear, robust, mature techniques
- Rotate the contours thru choice of redshift span
- Narrow the contours thru systematics control
- Break degeneracies thru multiple probes

Optimize an Experiment

Optimization depends on the question asked.



Recall that physics divided into 2 classes: thawing and freezing.

TABLE I: Figures of merit vary for different circumstances. Case denotes the region where the true universe lies (blank meaning all points in phase space are equivalent). Goal denotes the science objective, e.g. distinction from Λ or between thawing and freezing classes.

Case	Goal	Figure of Merit
Blank	Anything	Area
Thawing	Λ	Long axis
Thawing	vs. Freezing	$\sim w_a$
Freezing	Λ	Short axis
Freezing	vs. Thawing	$\sim$ Short axis
Defect	Λ	w_p

Design an Experiment

How to design an experiment to explore dark energy?

- **Choose clear, robust, mature techniques**
- **Rotate the contours thru choice of redshift span**
- **Narrow the contours thru systematics control**
- **Break degeneracies thru multiple probes**

With a strong experiment, we can even test the framework of physics.