Dark Energy Inhomogeneity and SNAP
Eric Linder

The cosmological constant is, by definition as the vacuum ground state energy, constant in time and spatially homogeneous (following from Lorentz invariance). A different form of dark energy, with constant or varying equation of state, will be neither (with inhomogeneity following from the Equivalence Principle). To the extent that dark energy can be viewed as arising from a very light mass scalar field it will be smooth on scales below its Compton wavelength, with inhomogeneities only relevant above that.

This scale is greater than 3000 h^{-1}Mpc, as can be seen from the simple derivation in the Appendix and numerical evaluation (Ma et al. 1999, astro-ph/9906174). [The numerics are actually of the influence of quintessence on matter, not quintessence inhomogeneity per se.] So we should not expect to detect a signature of dark energy inhomogeneity even with a wide field deep survey.

Inhomogeneity would most likely show not in any direct detection of the dark energy component (which is not directly seen even homogeneously) but through its dynamical effects in term of its equation of state w and anisotropic influence on the distance-redshift relation. However this seems unlikely for several reasons.

First, far more precise CMB measurements of the distance to the last scattering surface through the location of the first acoustic peak l_A show no anisotropy. The inhomogeneity scale would roughly correspond to anisotropic variation in l_A along lines of sight separated by the horizon size at last scattering, $1°$. Precision maps of the CMB exist for many hundreds of square degrees.

Second, the amplitude of the inhomogeneities at horizon crossing is presumably set by inflation like other perturbations to 10^{-5} (and CMB constraints require this as well). Third, inhomogeneities cannot grow until they are within the horizon and the universe is in a dark energy dominated phase, which did not occur until very recently (redshift less than one). Thus present day inhomogeneities are not expected to be substantial.

In addition, there are definite advantages for a SNAP sky survey strategy covering contiguous regions around the north and south ecliptic caps and disadvantages for a more diffuse map. From a science viewpoint, note that an anisotropy test can be applied between the north and south regions. There is no reason to believe that a $180°$ "dipole" separation between fields causes any problem. Second, the interesting details of weak lensing science that we will be concerned with by the time SNAP flies (nongaussianity, large scale dark matter map) are improved by having a contiguous, wide area field rather than small patches. From a space mission viewpoint, pointing all over the sky introduces difficulties with spacecraft motion, sun shielding, galactic extinction, etc.

In summary, the current SNAP survey field geometry offers superior science to spreading the fields over the sky.

Appendix. Derivation of Quintessence Inhomogeneity Scale

The time evolution of the quintessence field Q follows a Klein-Gordon equation, really just Newton's second law:

$$\ddot{Q} - \nabla^2 Q + 3H\dot{Q} = -V_{,Q} \ , \tag{1}$$

where H is the Hubble parameter of the universal expansion, creating a Hubble drag friction term, V is the potential energy of the field, and $_{,Q}$ denotes differentiation with respect to Q. Perturbing this we find

$$\delta\ddot{Q} + 3H\delta\dot{Q} + (k^2 + V_{,QQ})\delta Q = 0, \tag{2}$$

where we have switched to Fourier space, with k the comoving wavenumber. In fact, there will also be a source term on the right hand side, corresponding to inhomogeneities in the matter component, but this does not affect our results.

Just as in conventional matter perturbation theory, the spatial derivative term, usually written in terms of the sound speed c_s^2, affects the growth of perturbations. When that term is important growth is cut off due to pressure effects and this defines the Jeans length. So for growth we require $k^2 \ll V_{,QQ}$, defining a region

$$\lambda \gg \lambda_g = 2\pi/\sqrt{V_{,QQ}} \, , \tag{3}$$

where growth, and hence inhomogeneity, is possible.

We relate V to the quintessence energy density and pressure by

$$V = \frac{1}{2}(\rho_w - p_w) = \frac{1}{2}(1-w)\rho_w, \tag{4}$$

using the equation of state $p_w = w\rho_w$. Also note we can convert field derivatives to time derivatives by $V_{,Q} = \dot{V}/\dot{Q}$ and write $\dot{Q}^2 = (1+w)\rho_w$ from (twice) the kinetic energy. Thus

$$\rho_{w,Q} = -3H(1+w)\rho_w \, \dot{Q}^{-1} = -3H(1+w)^{1/2}\rho_w^{1/2}, \tag{5}$$

using the continuity Friedmann equation, and (assuming w constant)

$$\begin{aligned} \rho_{w,QQ} &= (1+w)^{-1/2}\rho_w^{-1/2}\dot{\rho}_{w,Q} = -3\rho_w^{-1/2}\frac{d}{dt}(H\rho_w^{1/2}) \\ &= -3[\dot{H} - (3/2)(1+w)H^2] \\ &= 12\pi\,(2\rho + p + w\rho). \end{aligned} \tag{6}$$

Note that these are now total densities and pressures.

Using eqs. (3) and (4) with a flat matter plus dark energy universe

$$\begin{aligned} \lambda_g &= \frac{4\pi}{3}H^{-1}\left[(1-w)(2+2w-w\Omega_m(z))\right]^{-1/2} \\ &\approx 10000\left(\frac{H_0}{H}\right)\, h^{-1}\text{Mpc}, \end{aligned}$$

an even larger scale than found by Ma et al. for the quintessence influence on the matter transfer function.