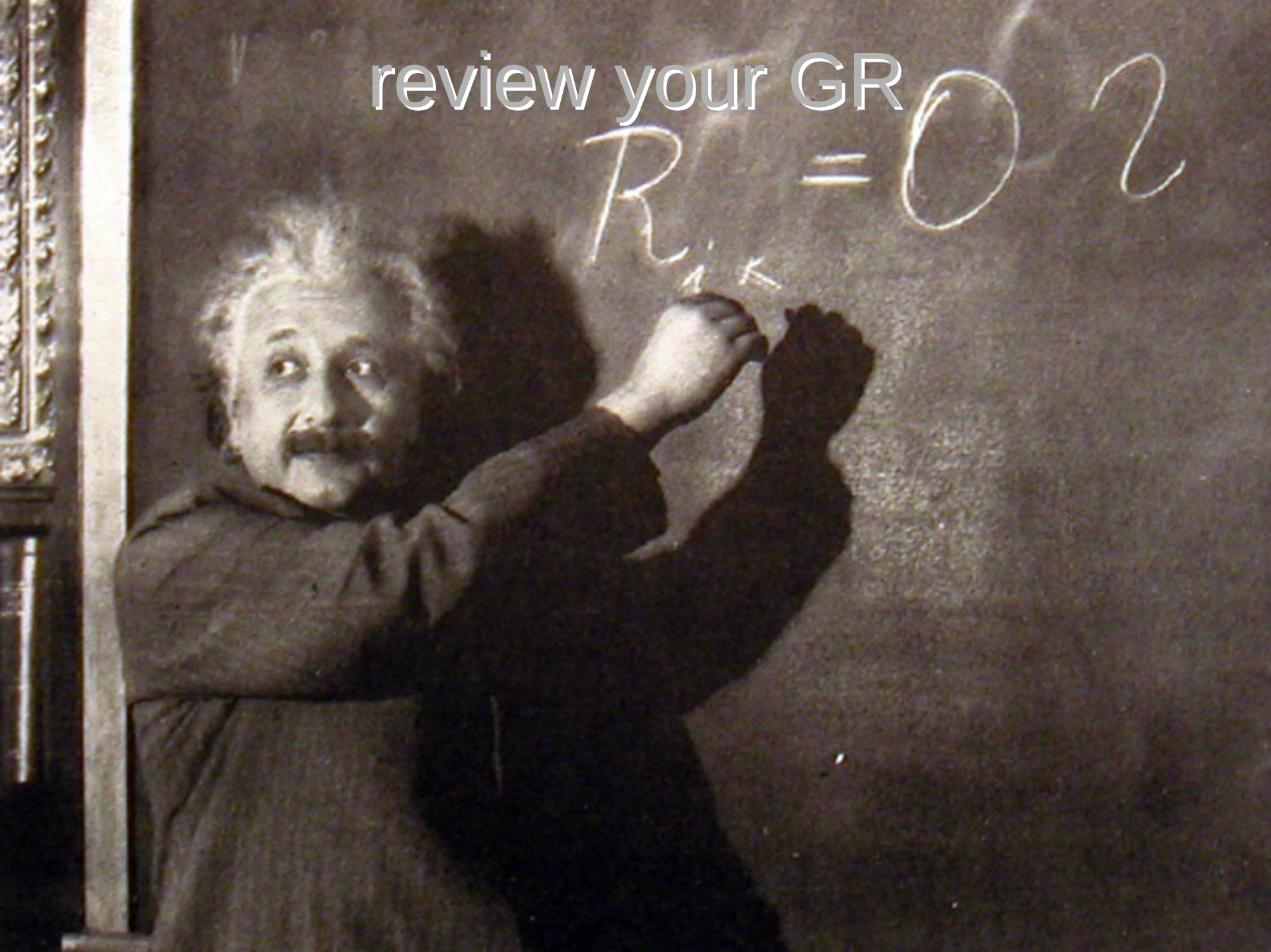
dark energy: cosmological challenge of the millennium

T. Padmanabhan

the paper

- parameters for standard cosmology
- geodesic acceleration
- fine tuning problem of Λ
- evolving scalar fields… not any better
- a universe with Λ… may not be so bad
- what could Λ possibly be?

review your GR



review your GR

$$R_{ab} - \frac{1}{2} g_{ab} R + \Lambda g_{ab} = 8\pi G T_{ab}$$

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

$$T_{ab} = \mathrm{diag}[\rho(t), -p(t), -p(t), -p(t)]$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

$$p = p(\rho) = w\rho$$

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p) = -3\frac{\dot{a}}{a}\rho(1 + w)$$

parameters of standard cosmology

CMB+SNeIa+LSS+BBN+…

for h~0.7

Ω_{DE}	~0.7
Ω_{DM}	~0.26
Ω_{B}	~0.04
Ω_{R}	~5.e-5
A	~(40Mpc)^4
n	~1

geodesic acceleration

spatial part of geodesic acceleration satisfies

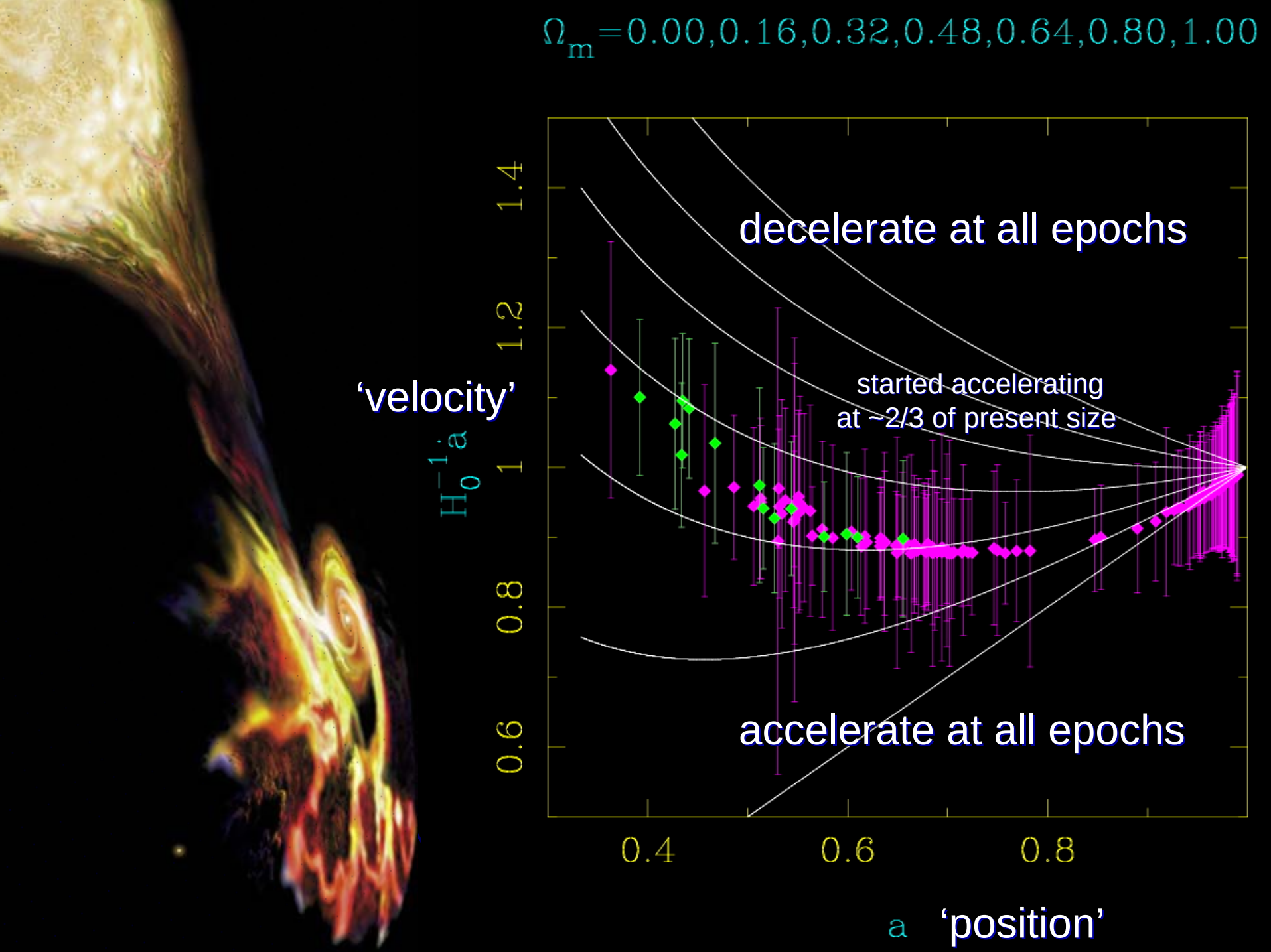
$$\nabla \cdot g = -4\pi G(\rho + 3p)$$

gravity attractive $1 + 3w > 0$

gravity repulsive $1 + 3w < 0$

- cosmological constant $w = -1 \quad \rho = -p = constant$

- dark energy with sufficiently negative pressure will accelerate the expansion, once it starts dominating over matter


Ω_m=0.00,0.16,0.32,0.48,0.64,0.80,1.00
decelerate at all epochs
started accelerating
at ~2/3 of present size
accelerate at all epochs
'velocity'
$H_0^{-1}\dot{a}$
1.4
1.2
1
0.8
0.6
0.4
0.6
0.8
a
'position'

fine tuning problem

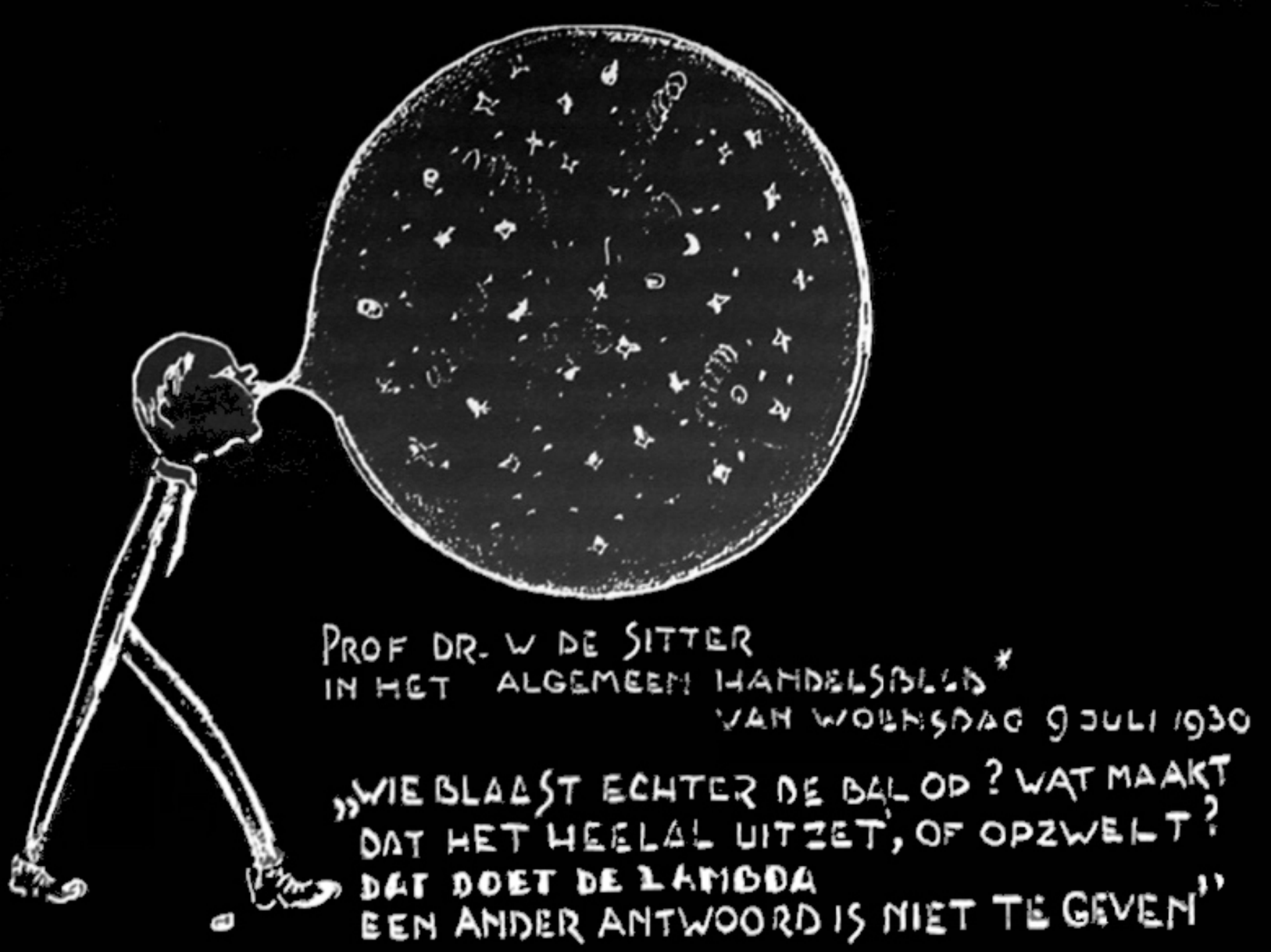
some fundamental scale lengths:

define dark energy density like matter/radiation density

Λ scale length: $L_\Lambda^2 = H_\Lambda^{-2} \equiv \frac{3}{8\pi G \rho_{DE}}$

Planck scale length: $L_p^2 \equiv \frac{G\hbar}{c^3}$

$$\left(\frac{L_p}{L_\Lambda}\right)^2 < 10^{-123}$$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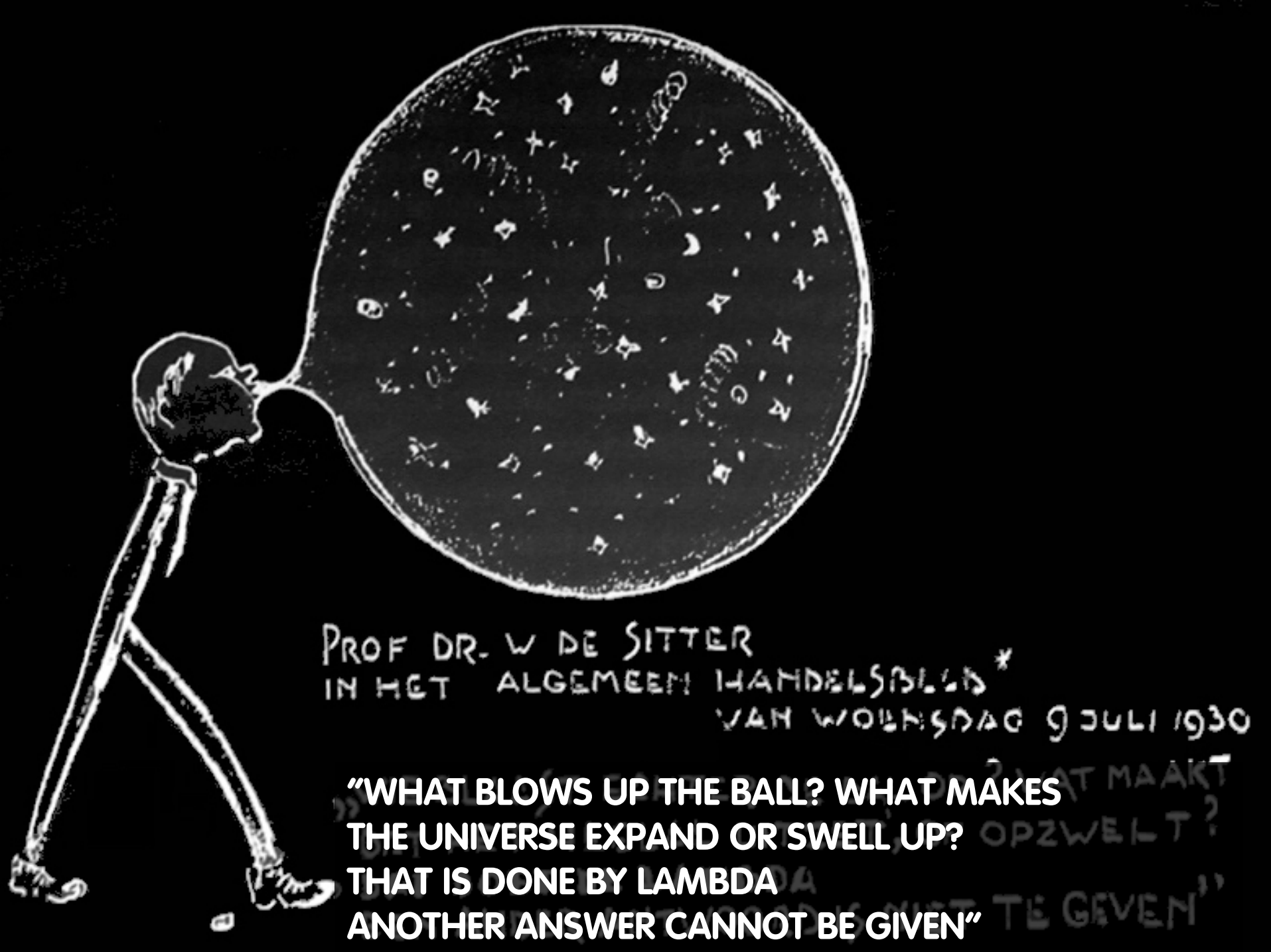
IN HET ALGEMEEN HANDELSBLAD*
VAN WOENSDAG 9 JULI 1930
„WIE BLAAST ECHTER DE BAL OP? WAT MAAKT
DAT HET HEELAL UITZET, OF OPZWELT?
DAT DOET DE LAMBDA
EEN ANDER ANTWOORD IS NIET TE GEVEN"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evolving scalar fields

scalar field φ, with suitably chosen potential V(φ)
to make vacuum energy vary with time

quintessence

$$\mathcal{L} = \frac{1}{2}\partial_a\phi\partial^a\phi - V(\phi)$$

natural generalization for non-relativistic particle

tachyonic

$$\mathcal{L} = -V(\phi)[1 - \partial_a\phi\partial^a\phi]^{1/2}$$

natural generalization for relativistic particle

evolving scalar fields

when the Lagragians act as sources
in a Friedmann universe

quintessence

$$\rho = \frac{1}{2}\dot{\phi}^2 + V$$

$$p = \frac{1}{2}\dot{\phi}^2 - V$$

$$w = \frac{1 - (2V/\dot{\phi}^2)}{1 + (2V/\dot{\phi}^2)}$$

tachyonic

$$\begin{aligned} \rho &= \rho_\Lambda + \rho_{DM} \\ &= V\sqrt{1 - \partial^i\phi\partial_i\phi} + V\frac{\partial^i\phi\partial_i\phi}{\sqrt{1-\partial^i\phi\partial_i\phi}} \end{aligned}$$

$$\begin{aligned} p &= p_\Lambda + p_{DM} \\ &= -\rho_\Lambda + 0 \end{aligned}$$

$$w = \dot{\phi}^2 - 1$$

evolving scalar fields

- V(φ) is an undermined function
- for any known/assumed w(a) or H(a) can get V(a), φ (a) → V(φ)
- "easy" to model w(a) with a suitable V(φ)

general forms of V(a) and φ (a) :

quintessence

$$V(a) = \frac{1}{16\pi G} H(1-Q)\left[6H + 2aH' - \frac{aHQ'}{1-Q}\right]$$

$$\phi(a) = \left[\frac{1}{8\pi G}\right]^{1/2} \int \frac{da}{a}\left[aQ' - (1-Q)\frac{d\ln H^2}{d\ln a}\right]^{1/2}$$

tachyonic

$$V(a) = \frac{3H^2}{8\pi G}(1-Q)\left(1 + \frac{2}{3}\frac{aH'}{H} - \frac{aQ'}{3(1-Q)}\right)^{1/2}$$

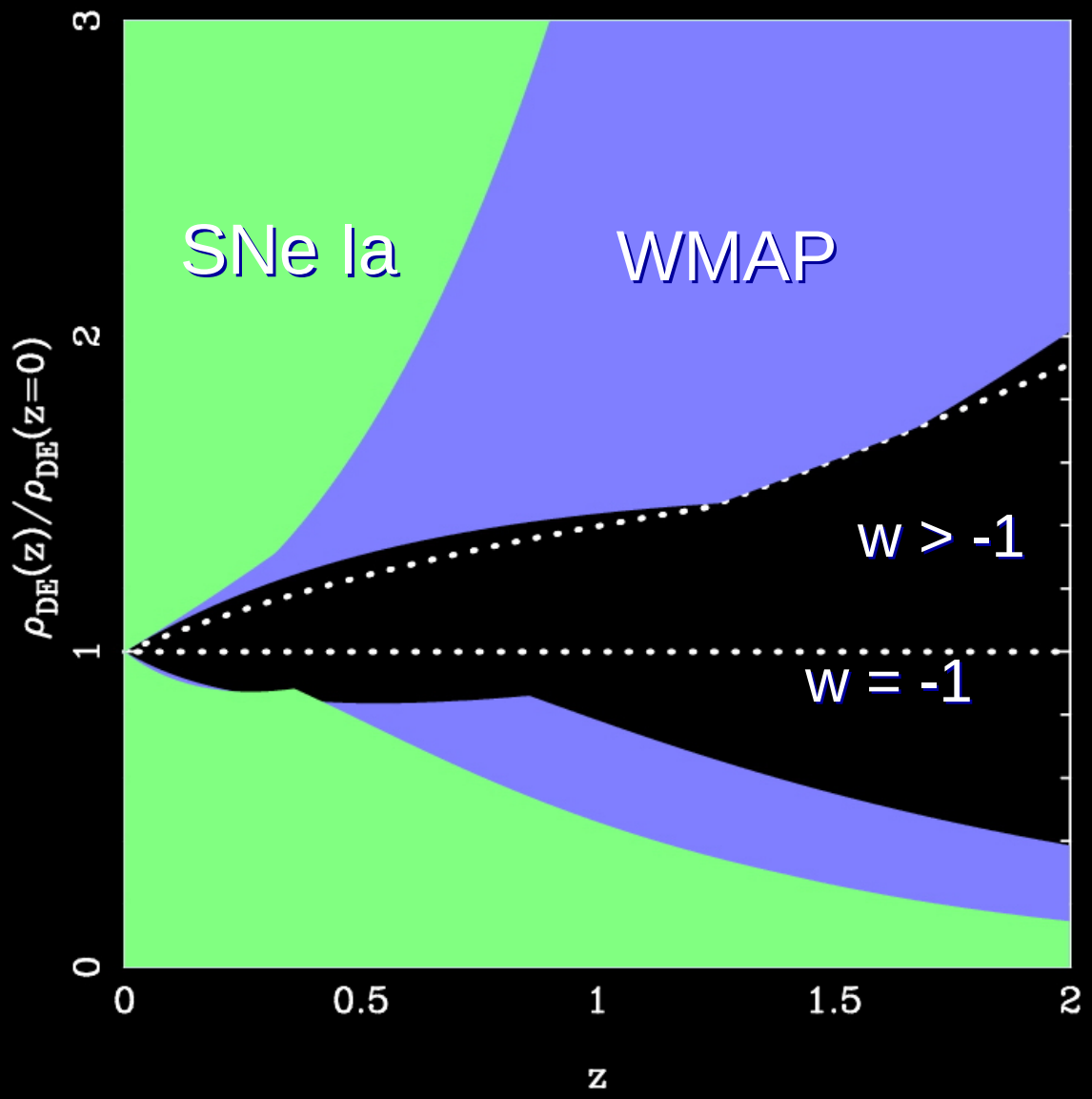
$$\phi(a) = \int \frac{da}{aH}\left(\frac{aQ'}{3(1-Q)} - \frac{2}{3}\frac{aH'}{H}\right)^{1/2}$$

where $Q(a) \equiv [8\pi G\rho_{\rm known}(a)/3H^2(a)]$

evolving scalar fields

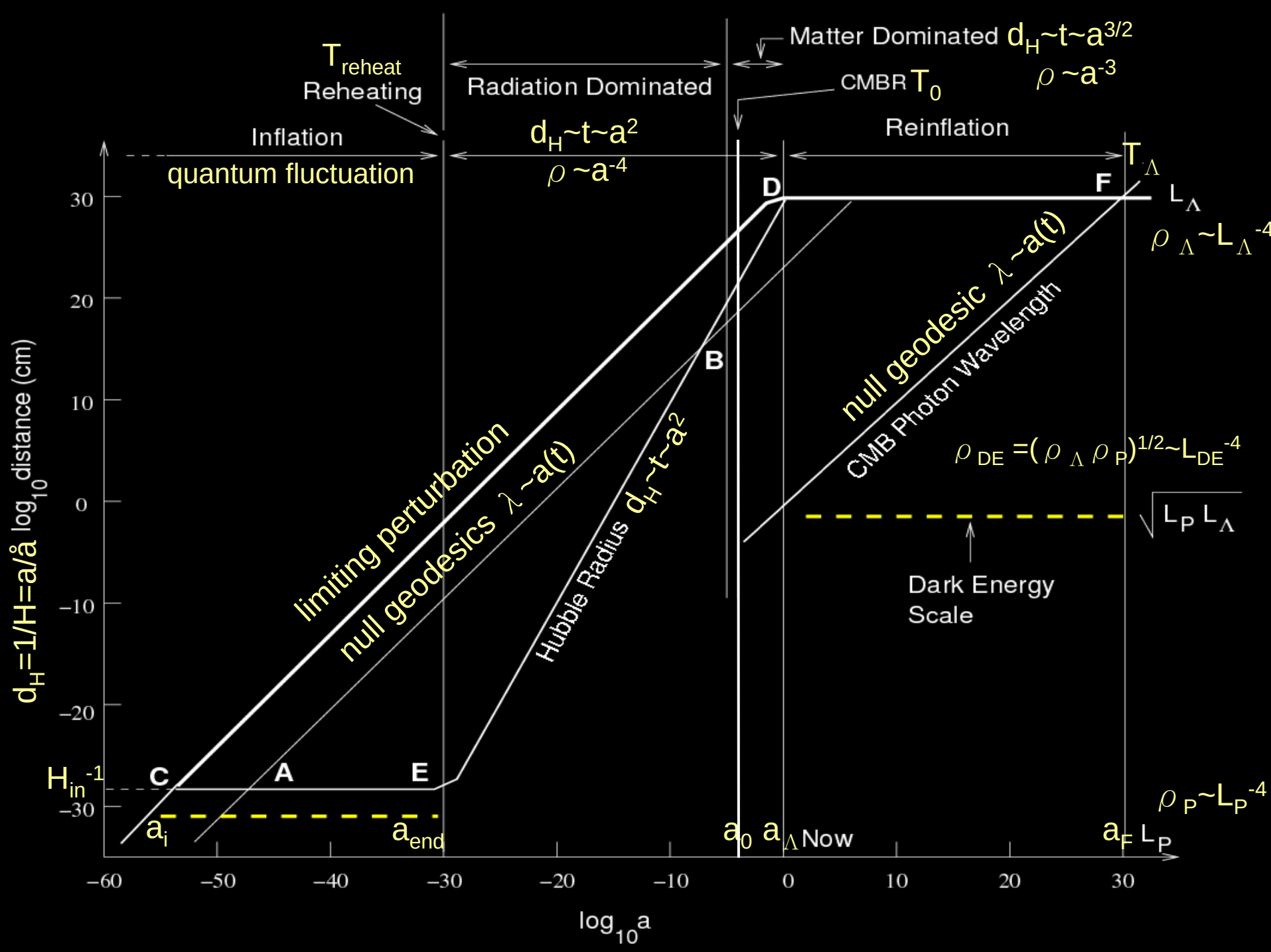
- completely lack predictive power: any form of a(t) can be modeled by a designer V(φ)
- even if w(a) is known, not possible to discriminate between quintessence/tachyonic
- V(φ) still require fine tuning of the parameters
- by and large, V(φ) in literature have no natural field theoretical justification
- all currently available observations are consistent with w = -1

observational constraints on w



a universe with Λ

- geometric structure consists of two deSitter phases: inflation (L_P) and re-inflation (L_Λ) + a radiation/matter dominated phase
- perfect cosmological principle: space + time translation invariance
- we don't occupy a special position in space nor a special moment in time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energy density of the universe

deSitter temperature:

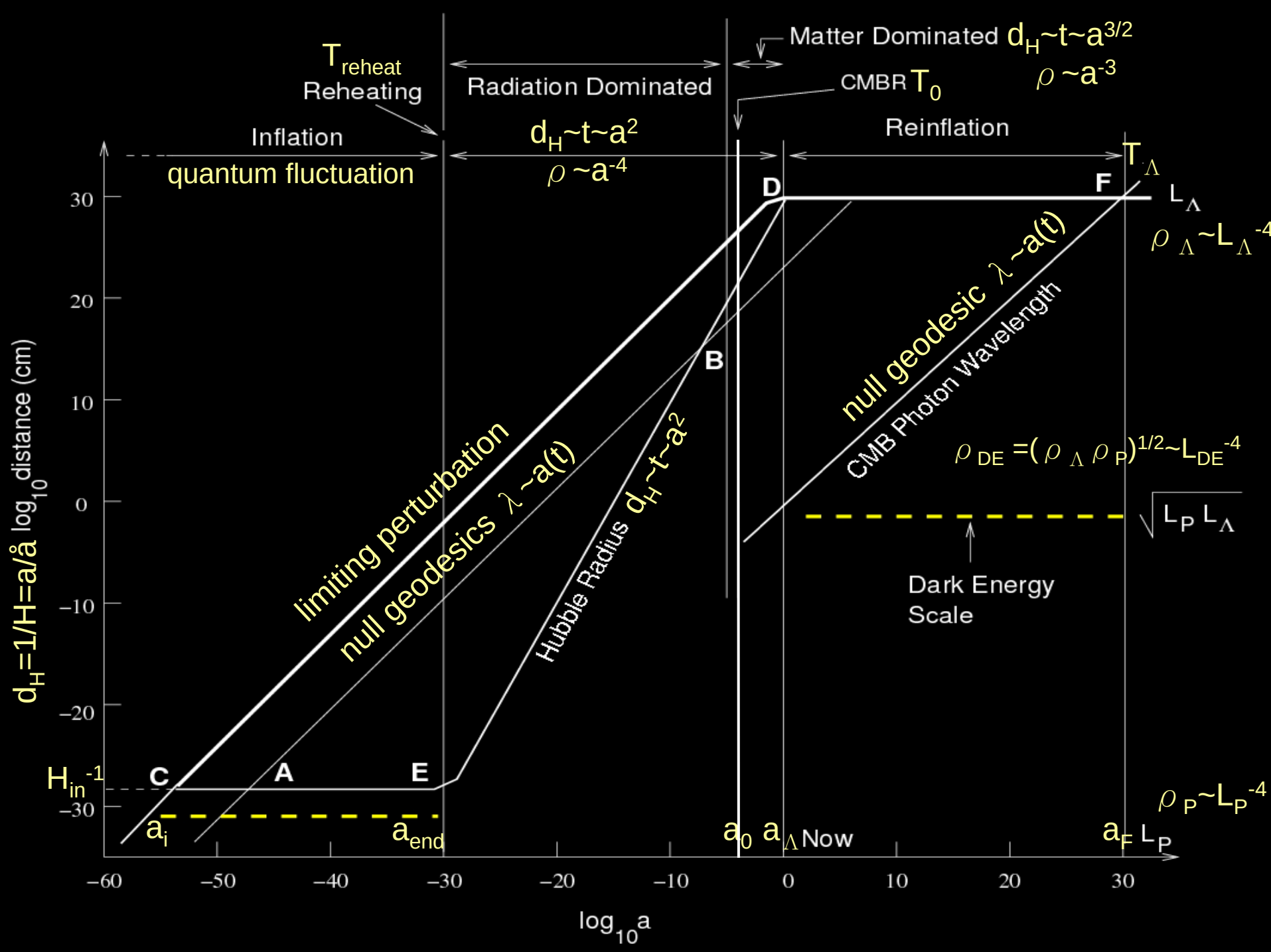
$$T_\Lambda = \frac{H_\Lambda}{2\pi} = \frac{1}{2\pi L_\Lambda} \qquad \rho_\Lambda \sim u \sim T^4 \sim L_\Lambda^{-4}$$

L_Λ defines the lowest possible energy density $\rho_\Lambda \sim L_\Lambda^{-4}$

L_P defines the highest possible energy density $\rho_P \sim L_P^{-4}$

dark energy density is the geometric mean

$$\rho_{DE} = \sqrt{\rho_\Lambda \rho_P} \qquad L_{DE} = \sqrt{L_\Lambda L_P}$$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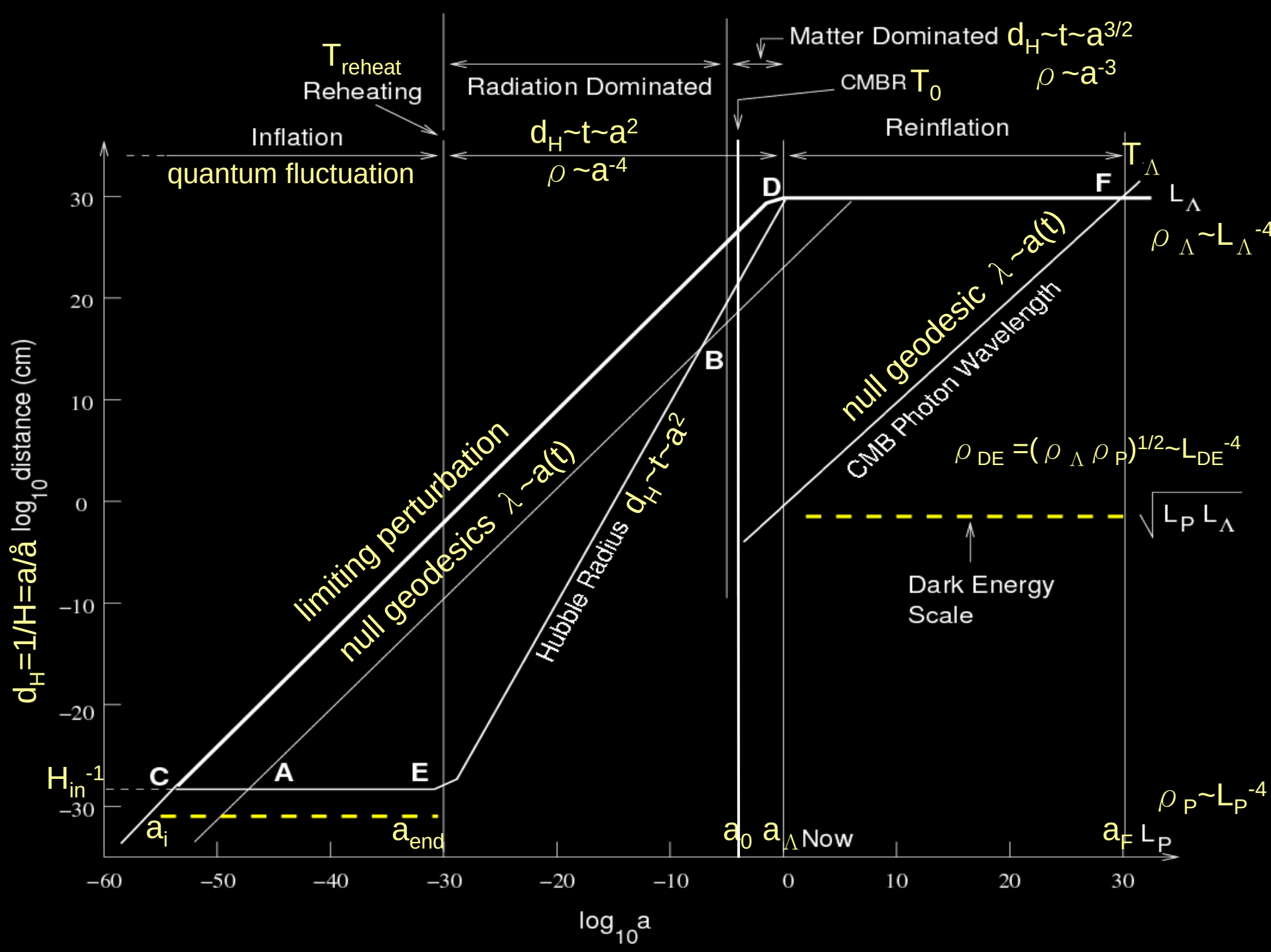
re-inflation phase

- deSitter phase:

$$\frac{\dot{a}}{a} = C, \quad \frac{da}{a} = C\,dt, \quad a \sim \exp(\mathrm{t/L})$$

- universe grows exponentially

- CMB photons
  - wavelength redshifted rapidly
  - temperature drops rapidly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re-inflation phase

- can estimate the dynamic range of re-inflation:
  - start when Λ takes over (end of matter dominated)

$$a \sim \rho^{-1/3}, \quad \frac{a_\Lambda}{a_0} \sim \left(\frac{\Omega_\Lambda}{\Omega_{DM}}\right)^{-1/3}$$

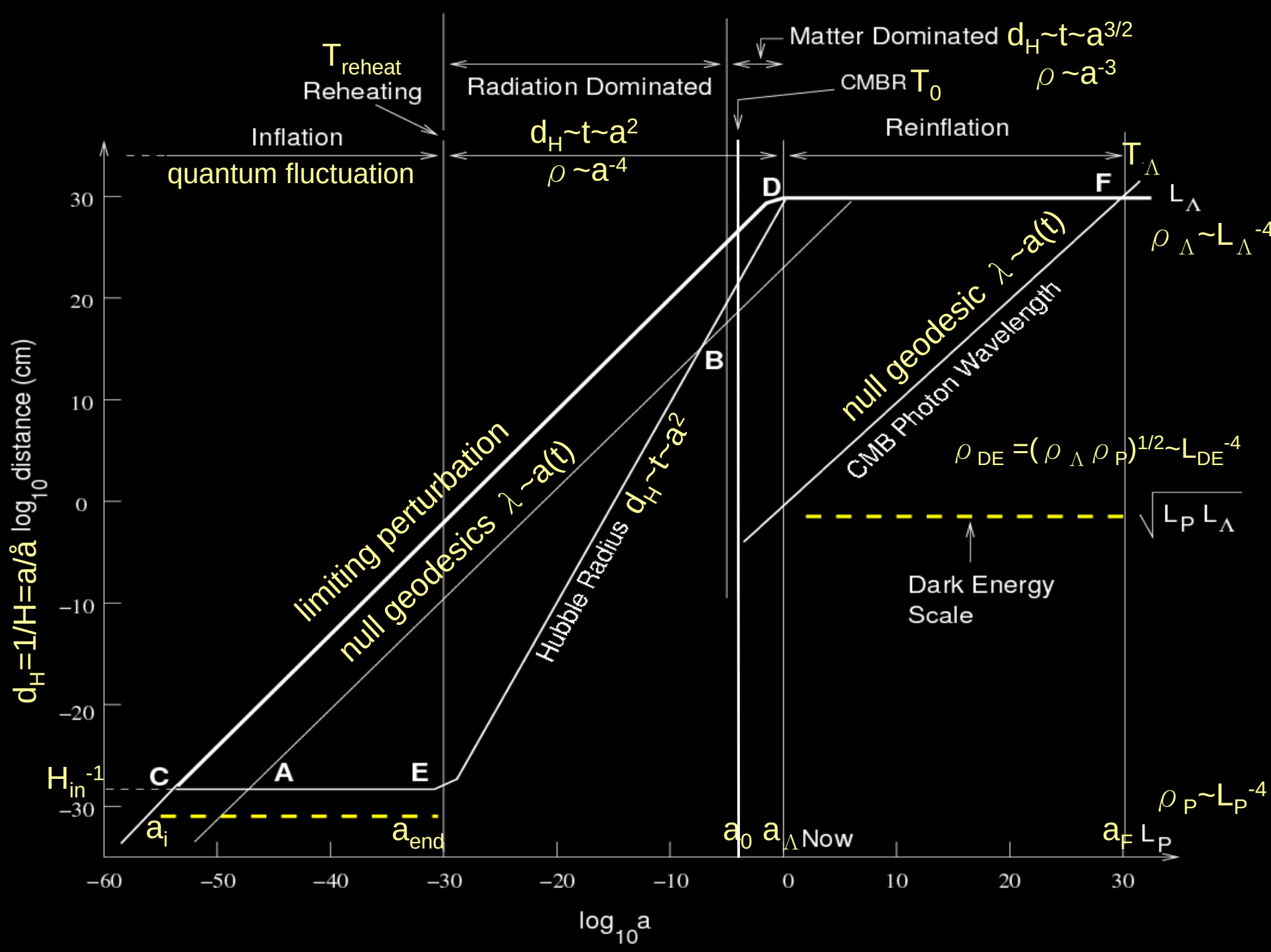
  - ends when CMB photon $\lambda \sim L_\Lambda$ or $T \sim T_\Lambda$

$$T \sim a^{-1}, \quad T_\Lambda \sim T_0 \left(\frac{a_0}{a_F}\right) \sim \frac{1}{2\pi L_\Lambda}$$

  - slap them together… a long long time from now

$$\frac{a_F}{a_\Lambda} \sim 2\pi T_0 L_\Lambda \left(\frac{\Omega_\Lambda}{\Omega_{DM}}\right)^{1/3} \sim 10^{30}$$

- universe dominated by vacuum thermal noise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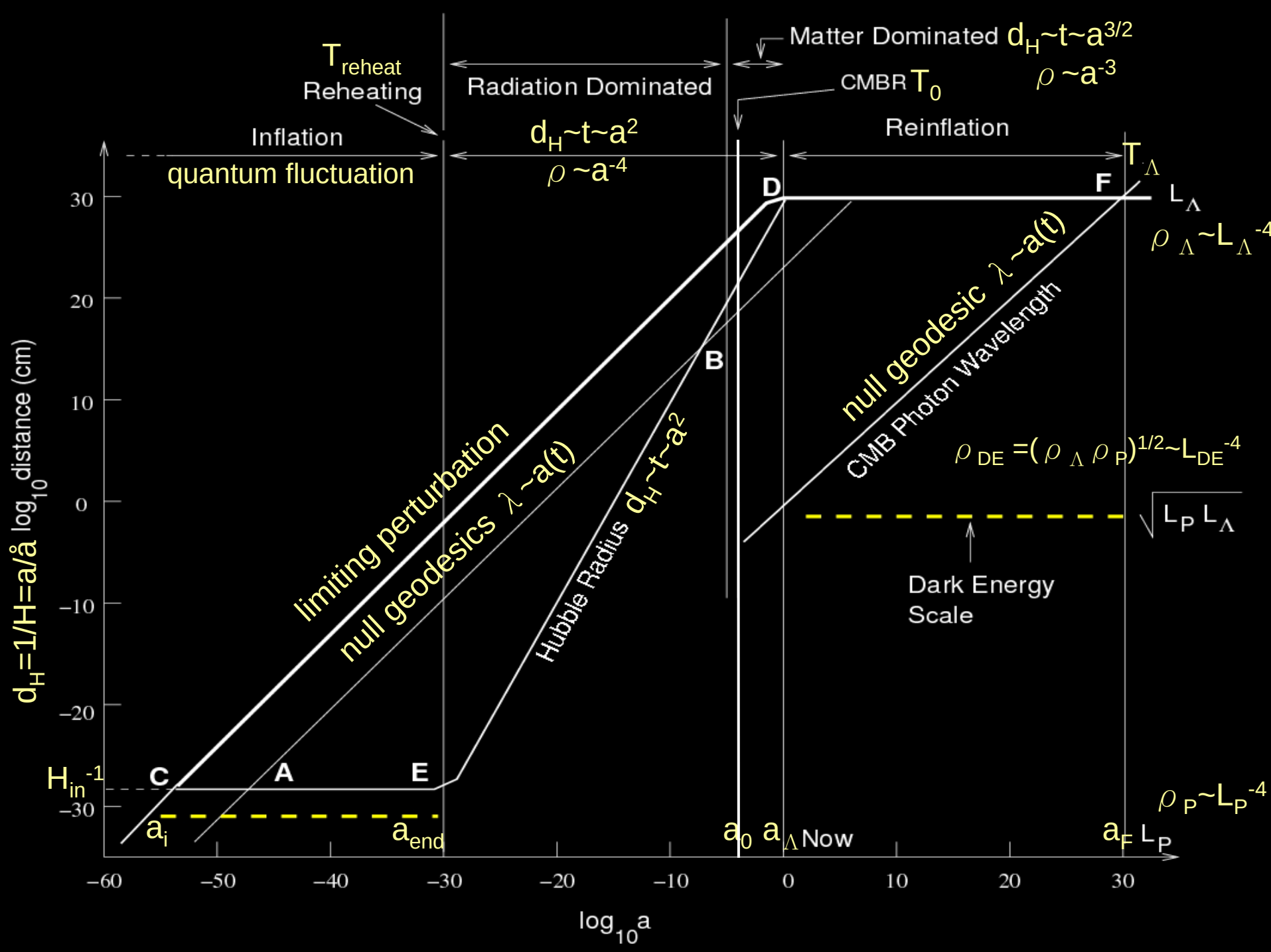
inflation phase

- hubble radius grows faster than the wavelength
- quantum fluctuations: seeds for structure formation
- perturbation
  - exits hubble radius during inflation
  - re-enters hubble radius some time later
  - if no re-inflation, all wavelengths re-enter
  - if re-inflation, natural bound on the duration of inflation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inflation phase

- can also estimate the dynamic range of inflation:

  start is bounded by limiting perturbation
  (just enters at the end of matter dominated phase)

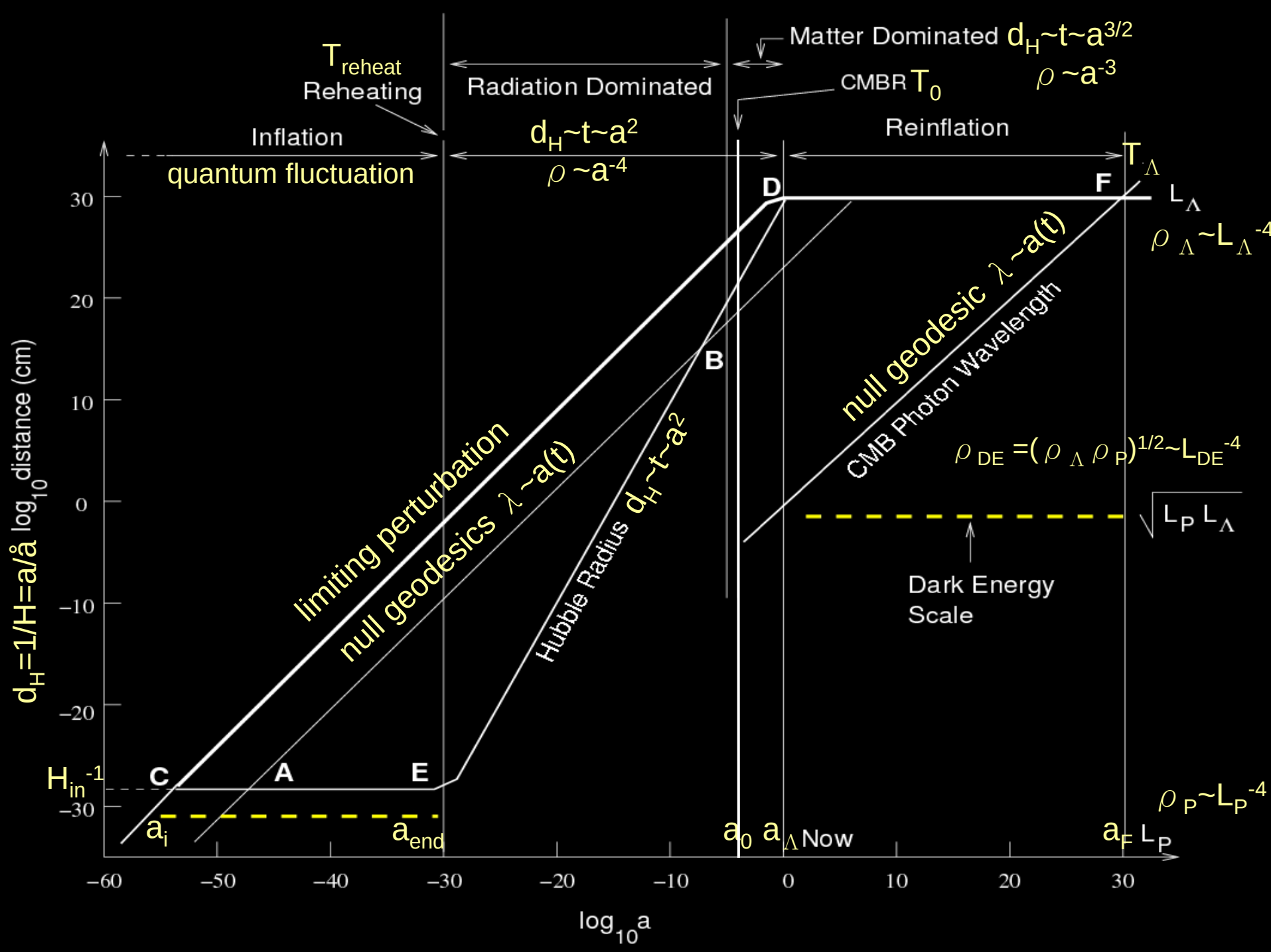
$$H_{in}\frac{a_\Lambda}{a_i} \sim L_\Lambda, \quad \frac{a_\Lambda}{a_0} \sim \left(\frac{\Omega_\Lambda}{\Omega_{DM}}\right)^{-1/3}$$

  ends when T~T_{reheat}

$$\frac{a_{end}}{a_0} \sim \frac{T_0}{T_{reheat}}$$

  slap them together…

$$\frac{a_{\rm end}}{a_i} \sim \left(\frac{T_0 L_\Lambda}{T_{\rm reheat} H_{in}^{-1}}\right)\left(\frac{\Omega_\Lambda}{\Omega_{DM}}\right)^{1/3} \sim \frac{(a_F/a_\Lambda)}{2\pi T_{\rm reheat} H_{in}^{-1}}$$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interpretation of Λ

3 approaches out of many:

- nonlinear correction term
- unimodular gravity
- scale dependence of vacuum energy

nonlinear correction term

$$\langle G_{ab}(g)\rangle \neq G_{ab}(\langle g\rangle)$$

$$G_{ab}(\langle g\rangle) = 8\pi G\langle T_{ab}\rangle + G_{ab}(\langle g\rangle) - \langle G_{ab}(g)\rangle$$

- T_{ab} of the real universe is inhomogeneous and anisotropic
- should average exact solution over large scale
- but practically, we average g_{ab} first, then solve
- difference can be characterized by a correction term
- if this term mimics cosmological constant at large scales, no need for dark energy
- one way to do this is using analytic approximations to nonlinear perturbations

unimodular gravity

…is it time yet? …did i type that out loud?

- eliminate from gravitational theory
  modes which couple to cosmological constant
- express metric in form $g_{ab} = f^2 q_{ab}$
  such that det(q) = 1, det(g) = f^4
- action functional for gravity:

$$A = \frac{1}{16\pi G}\int d^4x(R-2\Lambda)\sqrt{-g} = \frac{1}{16\pi G}\int d^4xR\sqrt{-g} - \frac{\Lambda}{8\pi G}\int d^4x f^4(x)$$

- Λ only couples f
- if q_{ab} only is varied, f kept constant,
  can obtain a form of a cosmological constant:

$$R_{ab} - \frac{1}{4}g_{ab}R = 8\pi G\left(T_{ab} - \frac{1}{4}g_{ab}T\right)$$

scale dependence of vacuum energy

- conventionally, the vacuum energy density ρ_{vac} based on evaluating zero point energy of quantum fields with an ultraviolet cutoff → too simplistic
- need to consider the fluctuations in ρ_{vac}
- Quantum theory:
  energy density, even of vacuum state,
  depends on the scale at which it is being probed
- example: vacuum state in the study of lattice vibrations is not the same as vacuum state of QED

scale dependence of vacuum energy

$$\rho_{\rm vac} = \frac{1}{L_P^4} + \frac{1}{L_P^4}\left(\frac{L_P}{L_\Lambda}\right)^2 + \frac{1}{L_P^4}\left(\frac{L_P}{L_\Lambda}\right)^4 + \cdots$$

bulk energy density

- needs to be renormalized away
  by process which we don't currently understand

scale dependence of vacuum energy

$$\rho_{\rm vac} = \frac{1}{L_P^4} + \frac{1}{L_P^4}\left(\frac{L_P}{L_\Lambda}\right)^2 + \frac{1}{L_P^4}\left(\frac{L_P}{L_\Lambda}\right)^4 + \cdots$$

bulk energy density

energy density $\rho_\Lambda \sim L_\Lambda^{-4}$

scale dependence of vacuum energy

$$\rho_{\rm vac} = \frac{1}{L_P^4} + \frac{1}{L_P^4}\left(\frac{L_P}{L_\Lambda}\right)^2 + \frac{1}{L_P^4}\left(\frac{L_P}{L_\Lambda}\right)^4 + \cdots$$

bulk energy density

energy density fluctuation

energy density $\rho_\Lambda \sim L_\Lambda^{-4}$

scale dependence of vacuum energy

$$\rho_{\rm vac} = \frac{1}{L_P^4} + \frac{1}{L_P^4}\left(\frac{L_P}{L_\Lambda}\right)^2 + \frac{1}{L_P^4}\left(\frac{L_P}{L_\Lambda}\right)^4 + \cdots$$

bulk energy density

energy density fluctuation

energy density $\rho_\Lambda \sim L_\Lambda^{-4}$

$$\Delta\rho_{\rm vac} \sim \frac{1}{L_P^2 L_\Lambda^2} \sim \frac{H_\Lambda^2}{G} \sim \rho_{DE}$$

for a universe with 2 length scale L_Λ and L_P vacuum fluctuations contribute energy density of correct order of magnitude

other alternatives to Λ

- almost all require:
  - introducing extra degrees of freedom in the form of scalar fields

    or
  - modifying gravity

    or
  - introducing higher dimensions

    and
  - fine tuning the potentials
- such approaches are unlikely to provide final solution at a fundamental level