

SCALAR FIELD PHYSICS

Lagrangian

$$\begin{aligned}\mathcal{L} &= (1/2)\partial^\mu\phi\partial_\mu\phi - V(\phi) \\ &\approx (1/2)\dot{\phi}^2 - V\end{aligned}$$

$$T^{ab} = \partial^a\phi\partial^b\phi - \mathcal{L}g^{ab}$$

$$\rho_\phi = (1/2)\dot{\phi}^2 + V \equiv K + V \quad ; \quad p_\phi = (1/2)\dot{\phi}^2 - V \equiv K - V$$

- Free Field $V=0$; $p = \rho$ \rightarrow Equation of state $w = +1$
- Slow Roll $K=0$; $p = -\rho$ \rightarrow Equation of state $w = -1$
- Coherent Oscillations $V = K$; $p = 0$ \rightarrow EOS $w = 0$

Evolution of Equation of State

$$\frac{d \ln w}{d \ln a} = 2 \frac{V}{V - K} \left[\frac{d \ln V}{d \ln a} + 6 \frac{K}{K + V} \right]$$

If K **not** $\ll V$ then w **not** ≈ -1 and

$w \neq \text{constant}$