Extragalactic Astronomy and Cosmology

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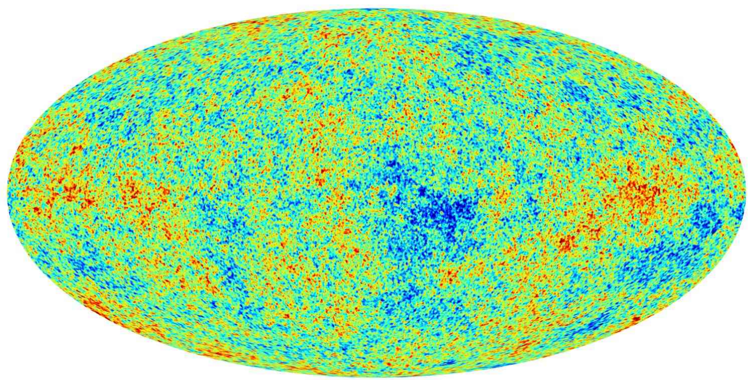

Know Your Universe

Vital Statistics of the Universe

$$R \approx 10^{28}\,\mathrm{cm} \qquad M \approx 10^{22}\,M_\odot$$

$$N_b \approx 10^{77} \qquad \rho \approx 10^{-29}\,\mathrm{g\ cm^{-3}}$$

$$T \approx 3\,\mathrm{K} \qquad t \approx 10^{10}\,\mathrm{y}$$

$$s \approx 10^{10}k \qquad p \approx 10^{-19}\,\mathrm{atm}$$

$$\text{Energy} \approx 0 \approx \text{Charge}$$

R = characteristic size of the universe

M = mass within the corresponding volume

N_b = number of baryons ("ordinary particles") in that volume

ρ = average mass/energy density in universe

T = characteristic background temperature of universe

t = age of universe

s = entropy per baryon of universe in units of Boltzmann's constant

p = average pressure of universe (in units of Earth's atmosphere)

Science of the Universe

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How do we approach a scientific description of the entire universe?

By using fundamental principles underlying physics – and testing them rigorously.

How do we describe the physics itself of the universe?

Einstein said the laws of physics, i.e. the *form* of physics, is the same everywhere in the universe.

This Principle of Equivalence is at the heart of cosmology.

Principle of Equivalence

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Theoretical form:

> *One can choose a coordinate system such that locally the form of the laws of physics is that in special relativity.*

Thus only need to understand special relativity and coordinate transformations to write laws of physics in form valid anywhere in universe.

Experimental form:

$$m_i/m_g = \text{constant}$$

Acceleration caused by forces (inertial mass m_i) is equivalent to properties of gravitation (gravitational mass m_g) – independent of internal properties of objects. Verified to $\mathcal{O}(10^{-12})$.

There is a much simpler way to remember this...

Equivalence Principle and Cosmology

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Einstein's Equivalence Principle:

Gravity = Curvature = Acceleration

Gravity is equivalent to the curvature of spacetime geometry, and determines the motions of particles along geodesics.

Forces (acceleration) change the motions of particles can be viewed as affecting spacetime geometry. Locally, acceleration is equivalent to gravity.

Equivalence Principle

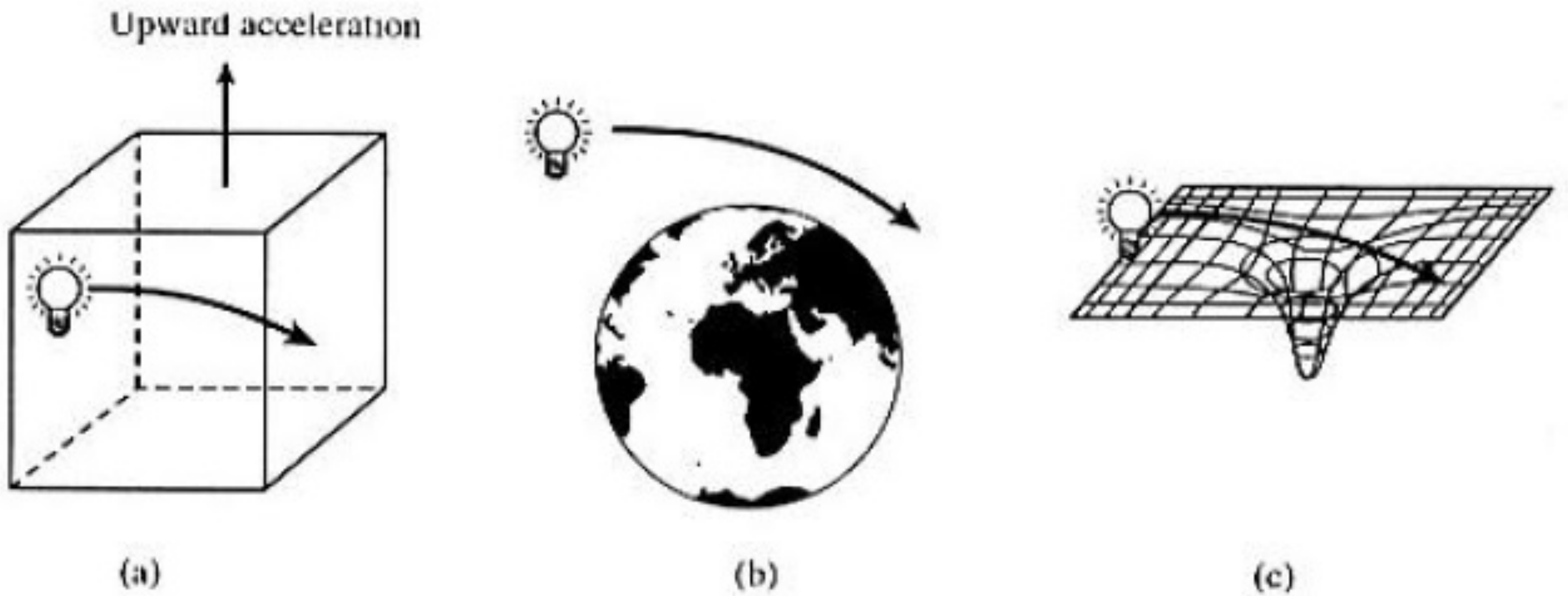
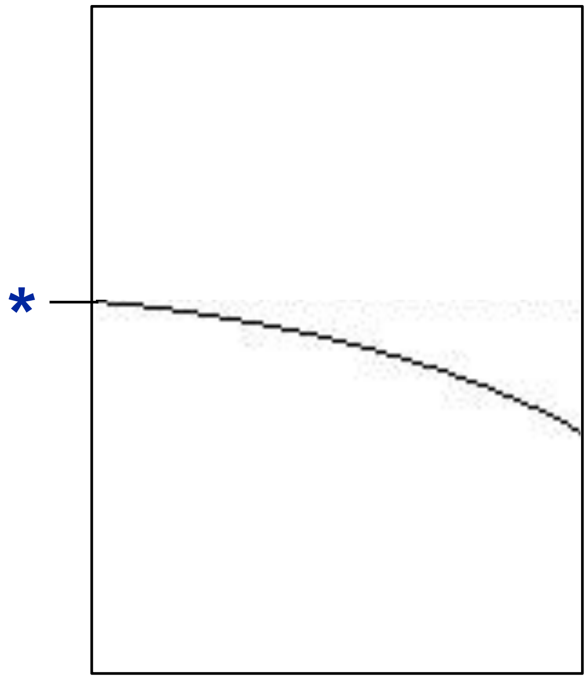
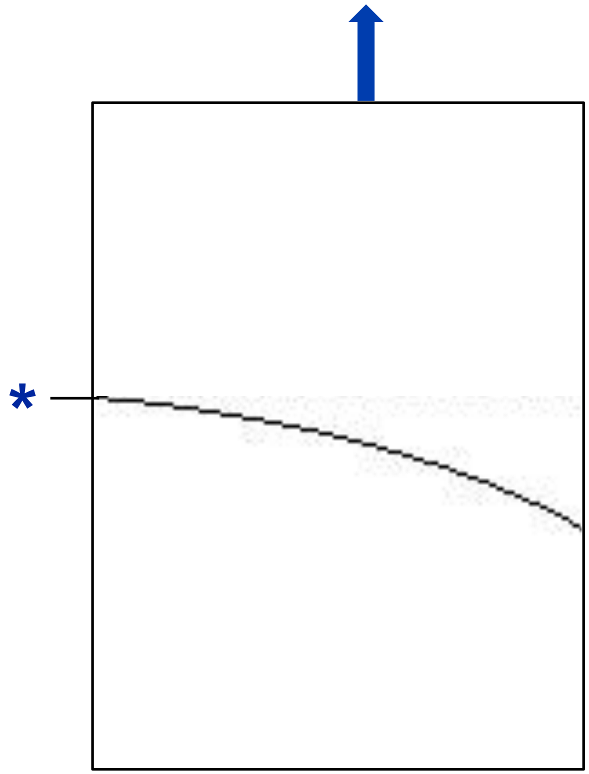


Figure 1.3 The motion of light equivalently interpreted as due to (a) acceleration in an elevator, (b) gravitational force of a mass, or (c) curvature of spacetime.

Acceleration = Gravity



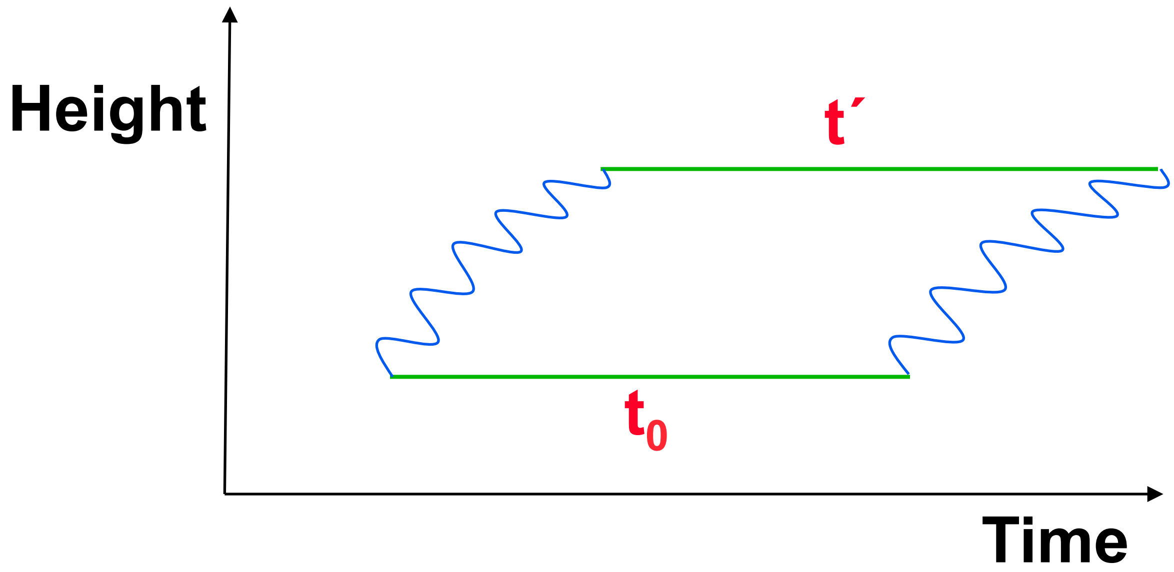


In the presence of gravity or of acceleration, light follows a curved path. Locally, they are equivalent.

Acceleration = Curvature

The Principle of Equivalence teaches that

Acceleration = Gravity = Curvature



Acceleration $\Rightarrow$ over time will get $v=gh/c$,
so $z = v/c = gh/c^2$ (gravitational redshift).

But, $t' \neq t_0$ $\Rightarrow$ parallel lines not parallel (curvature)!

Spacetime Curvature

Therefore we describe cosmology in a spacetime with curvature.

This is separate from whether *space* has curvature.

Mathematically, the Principle of Equivalence implies that the distance interval ds between events in spacetime must be given by a quadratic form of the coordinates dx^a:

$$ds^2 = g_{ab}\, dx^a\, dx^b$$

Gravity is the math and physics of the metric g_{ab}. Remarkably, you need to know almost no general relativity to do cosmology!

1 Minute General Relativity

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General relativity:

"Time and space and gravity have no separate existence from matter" – Albert Einstein

"Matter tells geometry how to curve, and geometry tells matter how to move" – John Wheeler

It can be developed through 1) Coordinate invariance and tensors, 2) Field theory of spin 2 fields, 3) Differential geometry.

$$G_{ab}[g_{ab}] \equiv R_{ab} - (1/2)Rg_{ab} \qquad \text{(spacetime)}$$

$$R_{ab} - (1/2)Rg_{ab} = 8\pi T_{ab} \qquad \text{(Einstein Field Eqs)}$$

$$\text{(particle physics)} \qquad 8\pi T_{ab} \equiv (2/\sqrt{-g})[\delta(\sqrt{-g}\mathcal{L})/\delta g^{ab}]$$

where T_{ab} is the energy-momentum tensor and $\mathcal{L}$ is the Lagrangian.

The Night Sky is Dark

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The most basic observation of the universe, accessible before modern cosmology, is that

The Night Sky is Dark

If you stare at a star, it appears very bright.
But overall the sky appears dark.

The intensity of light dies off as the inverse square power of distance R^{-2}.

However, the area of a shell at distance R, and so the number of stars, should go up as R^2.

Olbers' Paradox

$$F_{\mathrm{tot}} = \int dr\, F(r)\, \frac{dN(r)}{dr} = \int dr\, \frac{L}{4\pi r^2}\, 4\pi n r^2$$

$$= Ln \int dr \rightarrow \infty$$

The night sky should be infinitely bright!

This is known as Olbers' paradox (though it was earlier formulated by Halley and others).

Olbers' Paradox

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The resolution is the finiteness of the universe, in either space or time.

Since Newton it was realized that a spatially finite universe was unstable under gravity and should collapse.

Essentially the night sky is dark because the universe is finite in time. You can almost think of dark nights as direct evidence for the Big Bang! (or at least a finite age when stars existed).

You may sometimes hear that it's due to the expansion of the universe – that Halley could have discovered cosmic expansion. This is wrong, except when expansion gives a causal horizon (de Sitter space).

Our Isotropic Universe

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With modern cosmology, we can look beyond our galaxy and sample more of the universe.

By counting galaxies or quasars in different directions, we observe isotropy. With measurements of the cosmic microwave background (CMB) radiation, this is verified to the level of 10^{-5}.

Is the universe the same not only in all directions, but in all subvolumes – homogeneity?

Isotropy about 1 point does not guarantee homogeneity. 2D maps of the sky do not guarantee homogeneity.

Homogeneity

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With 3D redshift surveys (and other tricks), cosmology is starting to be able to test homogeneity directly.

Traditionally, we rely on the Copernican Principle, or Principle of Cosmic Modesty, that we are not at a preferred location, so isotropy about us implies isotropy everywhere, and hence homogeneity.

Isotropy + homogeneity gives a highly symmetric spacetime and makes the general relativity of our universe so simple we can use Newtonian analogs for the equations of motion.

Spacetime Geometry

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Homogeneity and isotropy determine the space to be maximally symmetric and the metric takes the Robertson-Walker form.

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

Spherical symmetry is obvious because the spatial sections involve two-spheres; for constant r the angular dependence is just $d\omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$

The key ingredients are

constant parameter k – spatial curvature,

function of time a(t) – scale (expansion) factor.

Spatial Curvature

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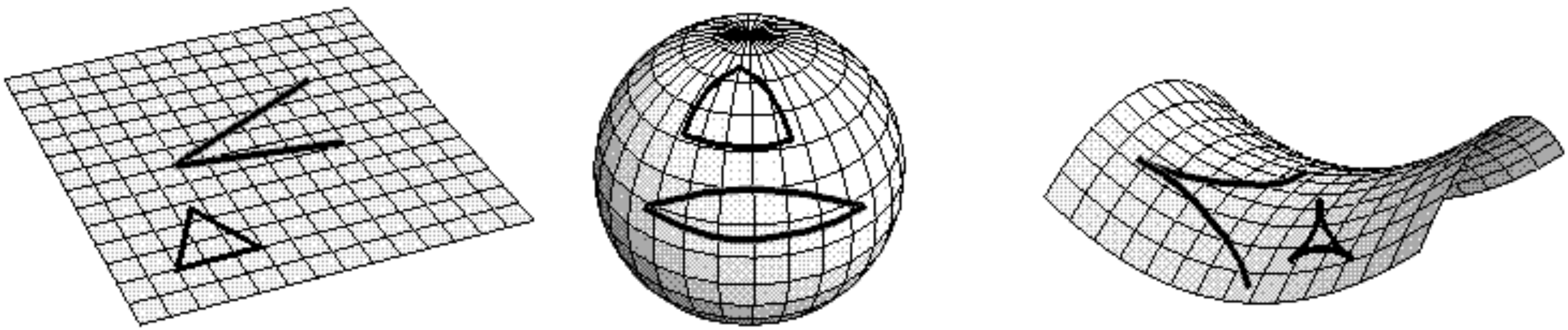
k isinverse square radius of curvature, $k=1/R_c^2$.
If $k=0$ then $R_c=\infty$ and space is flat.
$k>0$ indicates positive curvature (like a sphere),
$k<0$ negative curvature (like a hyperboloid/saddle).



k=0 **k>0** **k<0**

We can also choose to make r dimensionless (giving dimensions to a) and normalize $k=0, +1, -1$.

Cosmic Expansion

In front of the spatial part of the metric is the scale factor $a(t)$, scaling all distances. If a increases with time, this indicates cosmic expansion.

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

If r is dimensionful then a is dimensionless and we can normalize $a_{today}=a_0=1$. Cannot simultaneously normalize k and a!

2^{nd} derivatives of the metric g_{ab} $\quad ds^2 = g_{ab}\,dx^a dx^b$
form the Ricci tensor, determining spacetime curvature. This is proportional to $\ddot{a}$

Cosmic Expansion

Space flatness: k=0

Spacetime flatness: $\ddot{a} = 0$

The metric can be spatially flat (k=0) but the *spacetime* is curved if $\ddot{a} \neq 0$

This is exactly the Equivalence Principle:
Gravity = Curvature = Acceleration

All results coming directly from the metric (spacetime symmetries) are called kinematics.

We have not had to specify any laws of gravity!

(Results requiring force laws are called dynamics.)

Light Propagation

Kinematics can be read off from the metric. The most famous example is **redshift** and **cosmic expansion**.

Light signals travel on null geodesics (ds=0)

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

For a source at constant coordinates relative to the observer ("comoving") this says $\int dt/a$ = constant.

This is called the conformal time.

Light Propagation

Imagine a source pulsing with frequency ν~1/dt. The emission at t_e+dt_e is observed at t_o+dt_o. But

$$\int_{t_o}^{t_e} \frac{dt}{a} = \int_{t_o+dt_o}^{t_e+dt_e} \frac{dt}{a}$$

$$\Longrightarrow \frac{dt_e}{a(t_e)} - \frac{dt_o}{a(t_o)} = 0$$

Thus $$\frac{\nu_e}{\nu_o} = \frac{dt_o}{dt_e} = \frac{a_o}{a_e}$$

And we define the redshift

$$1+z = \frac{\nu_e}{\nu_o} = \frac{\lambda_o}{\lambda_e} = \frac{a_o}{a_e}$$

Redshift

Redshift

$$1 + z = \frac{\nu_e}{\nu_o} = \frac{\lambda_o}{\lambda_e} = \frac{a_o}{a_e}$$

is a purely kinematic effect.

General formula for redshift is

$$1 + z = \frac{(g_{ab}u^a k^b)_e}{(g_{ab}u^a k^b)_o}$$

where u^a is source 4-velocity, k^b is photon 4-momentum

Think About: What else can affect redshift?

Cosmic Expansion

If we normalize to us as observers, taking $a_o=1$,

$$a = \frac{1}{1+z} \qquad z = a^{-1} - 1$$

We can rewrite the conformal time in terms of the (logarithmic) expansion rate

$$\int \frac{dt}{a} = \int da \frac{dt}{da} \frac{a}{a^2} = \int da^{-1} \frac{dt}{d\ln a} = \int \frac{dz}{H}$$

$$H(a(t)) = \frac{\dot{a}}{a}$$ is the **Hubble parameter**.

Hubble Parameter

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The Hubble parameter H counts the number of e-folds of expansion per unit time, or the time per e-fold of expansion.

It sets the scale for the age and size of the universe.

Its present value is called the Hubble constant H_0.

So far, including expansion and H, this has all been kinematics. Weyl did this all, including the linear "Hubble law" in 1921, eight years before Hubble.

Much of modern cosmology is figuring out H(a(t)) – this requires knowing the dynamics, i.e. the force laws of gravity.

$$H(a(t)) = \frac{\dot{a}}{a}$$

Equations of Motion

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$$G_{ab}[g_{ab}] \equiv R_{ab} - (1/2)Rg_{ab} \qquad \text{(spacetime)}$$

$$R_{ab} - (1/2)Rg_{ab} = 8\pi T_{ab} \qquad \text{(Einstein Field Eqs)}$$

$$\text{(particle physics)} \qquad 8\pi T_{ab} \equiv (2/\sqrt{-g})[\delta(\sqrt{-g}\mathcal{L})/\delta g^{ab}]$$

where T_{ab} is the energy-momentum tensor and $\mathcal{L}$ is the Lagrangian.

Homogeneity + Isotropy also say the energy-momentum tensor takes the perfect fluid form

$$T_{ab} = (\rho + p)u_a u_b + p g_{ab}$$

Energy-Momentum Tensor

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$$T_{ab} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

The energy-momentum tensor depends only on the energy density ρ and the pressure p, or the equation of state parameter $w=p/\rho$.

Essentially,

$$G_{ab}(a,k) = 8\pi T_{ab}(\rho, p)$$

The energy density contents and equation of state determine the dynamics of the universe.

Cosmological Framework

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Equivalence Principle

→ **Metric description of spacetime**

Homogeneity and Isotropy

→ **Metric is Robertson-Walker (a,k)**

→ **Energy-momentum has perfect fluid form (ρ,p)**

Gravitational Field Eqs (General Relativity)
+ Homogeneity and Isotropy

→ **Friedmann equations for evolution of spacetime**

Equations of State + Friedmann equations

→ **Evolution of energy densities**

First Principles of Cosmology

If you liked this approach to learning cosmology, look at