CMB Anisotropies: Determining Cosmological Parameters

- Large angle measurements probe spectral tilt $n - 1$ through SW effect and curvature through late time SW, but masked by cosmic variance.

- Astrophysics, i.e. secondary anisotropies, prominent on small angles.

- Cosmology best detected on medium scales through acoustic oscillations left from photon-baryon coupling.

Acoustic Oscillations

Baryon perturbations don't grow but oscillate like sound waves.

$$\delta_b \propto c_s^{1/4} e^{i \int k c_s \, d\eta}$$

Recall

$$\begin{aligned} c_s &\equiv \sqrt{\frac{dp}{d\rho}} \\ &= \sqrt{\frac{c^2}{3}} \left(1 + \frac{3}{4}\frac{\rho_b}{\rho_\gamma}\right)^{-1/2} = \frac{c}{\sqrt{3}} \left(1 + \frac{3}{4}\frac{1 + z_{b\gamma}}{1 + z}\right)^{-1/2} \end{aligned}$$

The change of c_s from its high redshift value of $c/\sqrt{3}$ is sometimes called **baryon loading**.

So the amplitude of the acoustic signal is sensitive to Ω_b and the Hubble constant h.

The largest possible wavelength is given by the sound horizon size at decoupling, so the first acoustic peak occurs at

$$
\begin{aligned}
l = k r_a(z_{dec}) &= 2\pi r_a/\lambda \\
&= \pi \frac{r_a(z_{dec})}{r_h(z_{dec})}
\end{aligned}
$$

where

$$
r_h(z) = a(t) \int_0^t c_s dt/a = (1+z)^{-1} \int_{1+z}^{\infty} dy\, \frac{c_s(y)}{H(y)}
$$

The acoustic peaks occur in a harmonic series, the possible wavelengths being integer divisions of the maximum, so

$$
l_{peak\, m} = m\, l_{max} \quad , \quad m = 1, 2, 3 \ldots
$$

Since z_{dec} is remarkably insensitive to cosmological parameters, other than the already determined T_γ, cosmology enters simply through the ratio of the sound horizon size to the angular distance.

The sound horizon r_h depends on:

- $H(z)$ at early times – influenced by z_{eq} so depends on Ω_r, i.e. number of neutrino species, and Ω_m (including nonbaryonic matter), and h.

- $c_s(z)$ – depends on baryon-photon ratio η, contribution of helium abundance Y (to relate n_b to Ω_b), and h.

The angular distance r_a depends on:

- $H(z)$ at later times – influenced by Ω_{tot}, individual components Ω_σ,

e.g. cosmological constant Λ, and h.

At asymptotically high redshift,

$$r_a \to 2H_0^{-1}\Omega_m^{-1}z^{-1}$$

$$r_h \to \frac{2}{\sqrt{3}}H_0^{-1}\Omega_m^{-1/2}z^{-3/2}$$

since baryon loading is negligible and $c_s \to 1/\sqrt{3}$. So

$$l_{\max} = \Delta l = \pi\sqrt{3}\Omega_m^{-1/2}z^{1/2}$$

$$\to 180\,\Omega_m^{-1/2}$$

using $z_{dec} = 1100$.

Thus the peak position and spacing probes the matter density.

A cosmological constant has little effect at high redshift so in a flat inflationary universe $(\Omega_m + \Omega_\Lambda = 1)$ r_h is unchanged but $r_a \propto \Omega_m^{-1/2}$ so $\Delta l \propto \Omega_m^0$.

Golden Rule:

The peak position and spacing probes the total density Ω_{tot}.

The peak amplitude probes the baryon density Ω_b and h.

The insensitivity to components, e.g. Ω_m or Λ, separately is called **degeneracy**. In fact, z_{dec} is not in the asymptotic regime so the degeneracy is somewhat lifted. The Golden Rule is really more a rule of thumb.

Accurate observation of many peaks does allow separate determination of cosmological ingredients, may even yield secondary parameters such as N_ν.

Figures from animations of C_l anisotropy dependence on parameters at

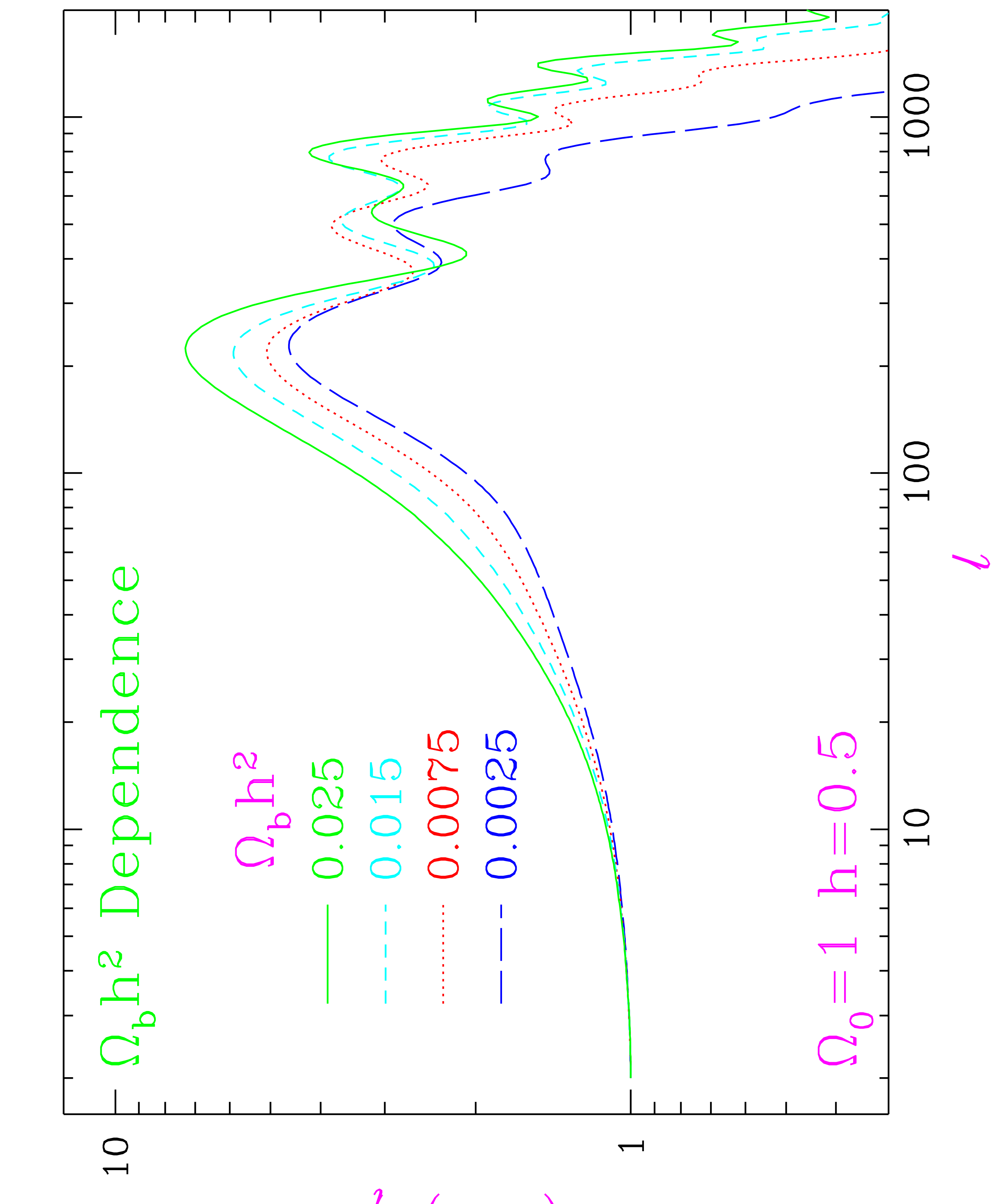
http://www.sns.ias.edu/˜whu/metaanim.html

Inflationary Parameters

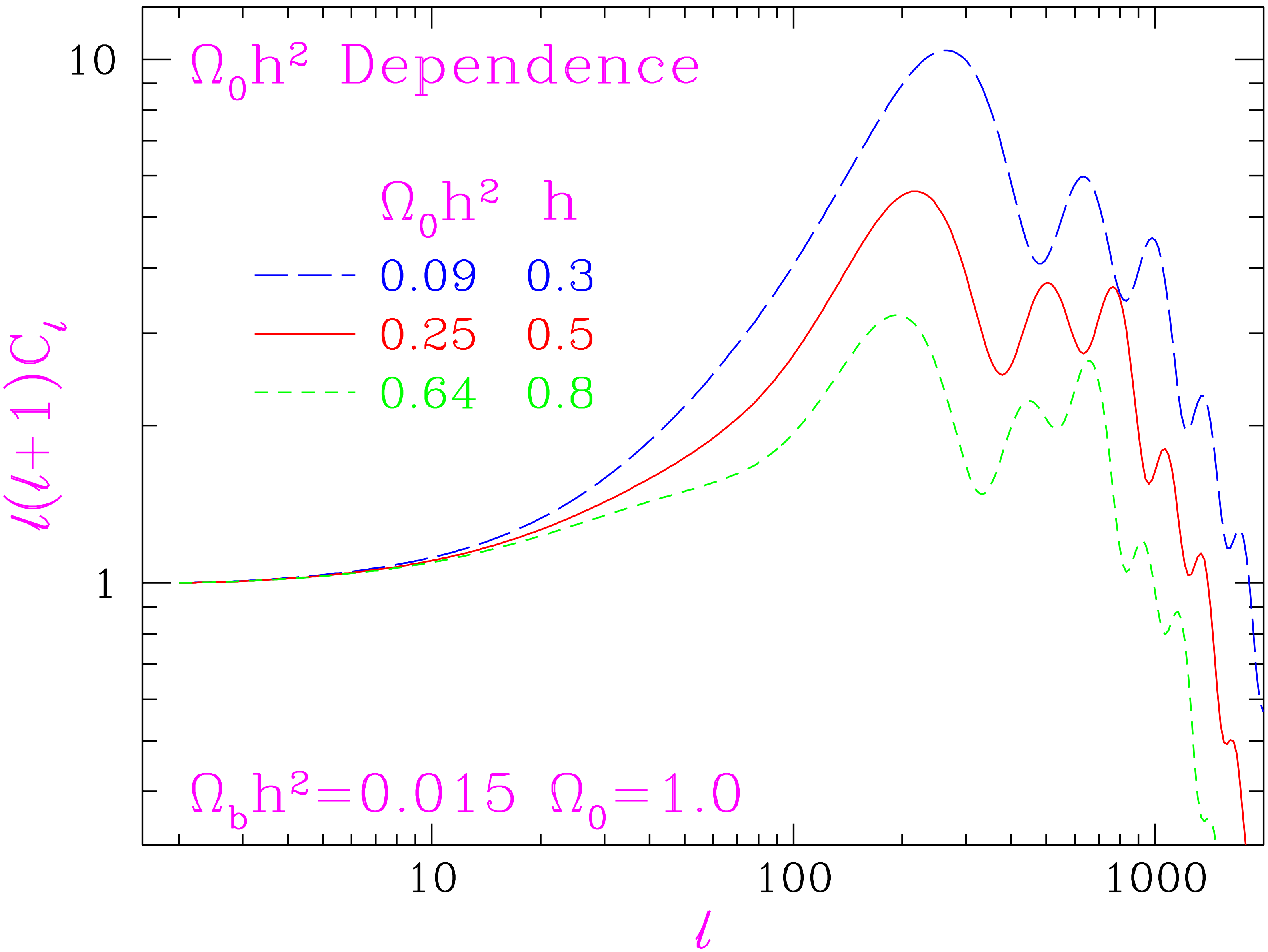
Most inflation theories predict n near unity, but with tilt.

DeSitter, or exponential, inflation predicts no tilt but passé.

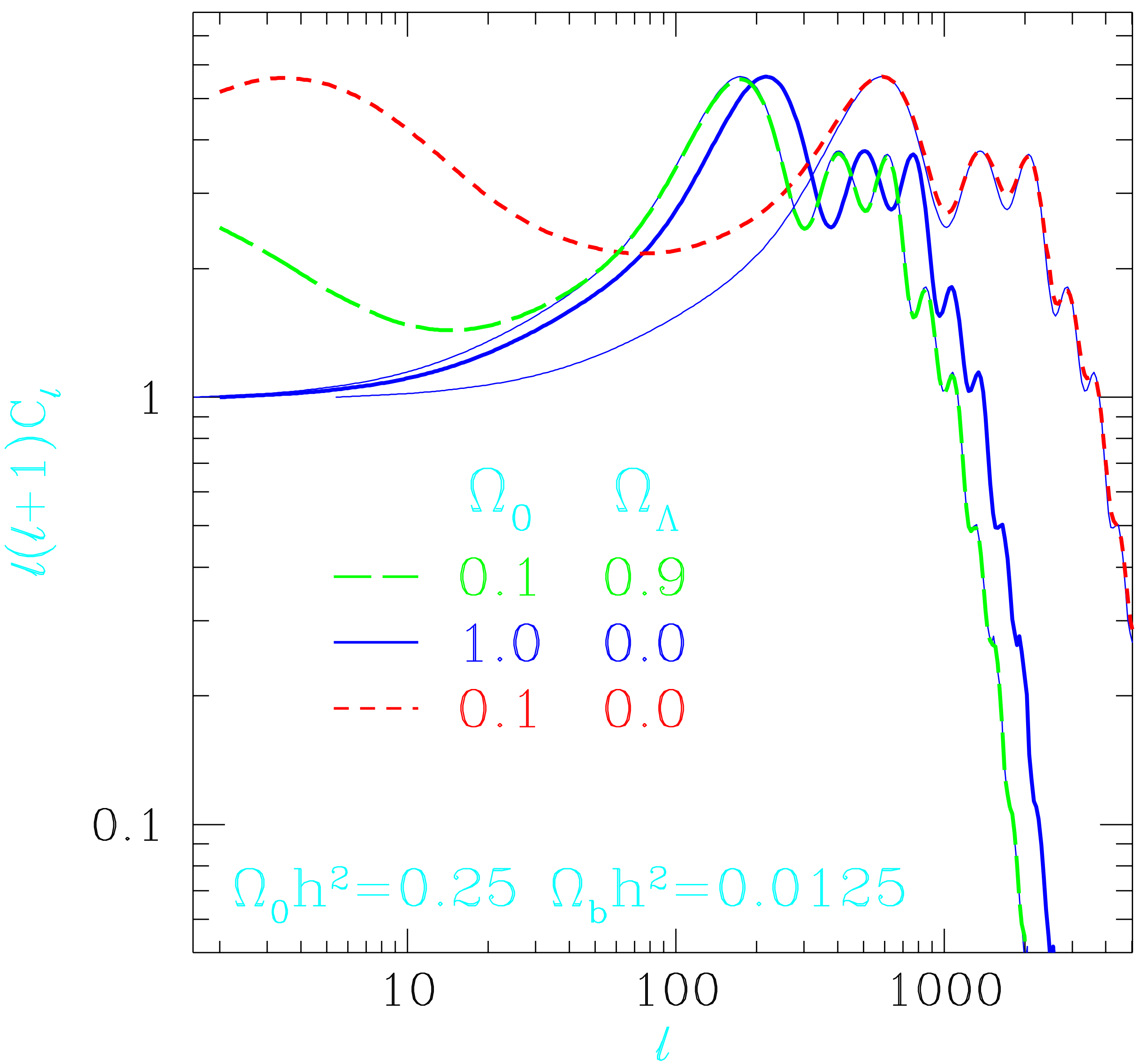
Simplest class of realistic inflation is **power law** inflation, where $a \propto t^p$,



$\Omega_b h^2$ Dependence
$\Omega_b h^2$
0.025
0.015
0.0075
0.0025
$\Omega_0=1$ h=0.5
10
100
1000
l
10
1
$l(l+1)C_l$



$\Omega_0 h^2$ Dependence
$\Omega_0 h^2$ h
0.09 0.3
0.25 0.5
0.64 0.8
$\Omega_b h^2 = 0.015$ $\Omega_0 = 1.0$
$l(l+1)C_l$
l
10
1
10
100
1000


$\ell(\ell+1)C_\ell$
ℓ
1
0.1
10
100
1000
Ω_0 Ω_Λ
0.1 0.9
1.0 0.0
0.1 0.0
$\Omega_0 h^2=0.25$ $\Omega_b h^2=0.0125$

$p > 1$. These also predict existence of additional, tensor perturbations or **gravitational waves**.

Key elements are **slow roll parameters** measuring shape of inflation potential $V(\phi)$

$$\epsilon \equiv \frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2$$
$$\eta \equiv \frac{1}{8\pi G}\left(\frac{V''}{V}\right)$$

where $' = d/d\phi$.

Scalar and tensor power

$$P_k \equiv |\delta_k^S|^2 \propto k^n$$

$$P_k^T \equiv |\delta_k^T|^2 \propto k^{n_T}$$

with indices

$$n = 1 - 6\epsilon + 2\eta$$

$$n_T = -2\epsilon$$

Ratio of tensor to scalar power

$$r = \frac{T}{S} \equiv \frac{C_l^T}{C_l^S} = 12.4\epsilon$$

Consistency condition for slow roll approximation gives

$$n = 1 - \frac{1}{6.2}\frac{T}{S} = \frac{p-3}{p-1}$$

where the last equality is for power law inflation.

So gravitational waves imply tilt.

To detect gravitational wave perturbation to CMB directly, likely to need CMB polarization measurements – next generation.

Test of entire paradigm for structure formation

- Is inflation origin of primordial perturbations?
    - Are perturbations adiabatic?
    - Are perturbations Gaussian?
    - What is form of the potential, i.e. GUT/Planck energy physics?
- Is gravitational instability the driving mechanism for structure formation?
- What is the dark matter?
- What are the values of the cosmological parameters?

Greatest leverage in probing universe through combination of CMB results with galaxy power spectrum, supernovae distance-redshift tests.

The truth is out there (and not far off)

	1997	BOOM/MAX	MAP*	Planck*
Ω	0.01 - 2	6%	18%	1%
Ω_b	$0.01h^{-2}$	30%	10%	0.7%
$\Lambda(\Omega_\Lambda)$	< 0.65	±0.10	±0.43	±0.05
Ω_ν	< 2	±0.25	±0.08	±0.03
t_0	12-18 Gyr	—	—	—
H_0	30-80 km/s/Mpc	10%	20%	2%
σ_8	0.5-0.6	30%	30%	10%
Q	20 ± 2 μK*	"	"	"
n_s	1.0 ± 0.5	30%	5%	1%
τ	0.01 - 1	±0.5	±0.2	±0.15
T_0	2.73 ± 0.01*	—	—	—
Y	0.2-0.25	10%	10%	7%
T/S	0.0 - 1	±1.6	±0.38	±0.09

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