**CMB Anisotropies: Statistics and Measurements**

Physical processes create a pattern of temperature anisotropies $\delta T/T\,(\vec{\theta})$ on the sky. In analogy to density fluctuations in space can form correlation functions and power spectra.

**Correlation Function:**

$$C(\theta) = \left\langle \frac{\delta T}{T}(\vec{\psi})\,\frac{\delta T}{T}(\vec{\psi}+\vec{\theta}) \right\rangle$$

$\sqrt{C(0)}$ is rms temperature anisotropy.

**Power Spectrum:**

Instead of Fourier decomposing into plane waves, for spherical sky use **spherical harmonics** $Y_{lm}$.

$$\begin{aligned}
\frac{\delta T}{T}(\vec{\psi}) &= \sum_{l,m} a_{lm} Y_{lm}(\vec{\psi}), \\
C(\theta) &= \frac{1}{4\pi} \sum_{l} \sum_{m=-l}^{m=+l} \langle |a_{lm}|^2 \rangle P_l(\cos\theta) \\
&= \frac{1}{4\pi} \sum_{l} (2l+1)\, C_l P_l(\cos\theta)
\end{aligned}$$

where orthogonality of spherical harmonics gives Legendre polynomial $P_l$:

$$Y_{lm}(\vec{\psi})\, Y^*_{l'm'}(\vec{\psi}+\vec{\theta}) = P_l(\cos\theta)\,\delta_{ll'}\,\delta_{mm'}\,.$$

The transform partner of the angular separation $\theta$ is the **multipole** $l$.

**Ensemble average power** $C_l = \langle |a_{lm}|^2 \rangle$. We observe only one statistical **realization** not an ensemble, but do observe many angular patches. Ensemble average equivalent to angular average if **ergodic theorem** holds. For large angles there are few patches so poor statistics and unavoidable **cosmic variance** $\delta C_l$.

$C_0$, the monopole power, is just poor background definition $\langle \delta T \rangle \neq 0$.

$C_1$, the dipole power, is subtracted because it is due to our motion, not primordial anisotropies. Know this because on large scales (small $l$) Sachs-Wolfe dominates and predicts $C_1 = 3C_2$. But observationally $C_2 \ll C_1$: 99% of $C_1^{1/2}$ is due to kinematics.

First primordial anisotropy term is quadrupole, $C_2$. Not detected until 1992. Sachs-Wolfe implies

$$C_l = C_2 \frac{\Gamma[l + (n-1)/2]}{\Gamma[l + (5-n)/2]} \frac{\Gamma[(9-n)/2]}{\Gamma[(3+n)/2]} \to \frac{6}{l(l+1)} C_2$$

where $n$ is density power spectrum index and the limit is for $n = 1$.

Measure of broad band power, in $dl/l$,

$$T_l^2 = (1/2\pi)\, l(l+1) C_l.$$

This is *flat* for the Sachs-Wolfe effect of a scale invariant spectrum. Deviation from flatness can be direct probe of deviation of $n$ from unity; hence $n-1$ is called **tilt**.

### Angular Scales, Comoving Wavelengths, and Multipoles

Translate comoving wavelengths of *perturbations* into angular scales or multipoles of *anisotropies* by

$$\theta = \frac{\lambda}{(1+z)r_a(z)} \to 0.59' \left(\frac{\lambda}{h^{-1}\mathrm{Mpc}}\right)$$
$$l = kr_c(z) \approx \frac{1}{\theta} = 57\left(\frac{\theta}{1^\circ}\right)^{-1} \to 5800\left(\frac{\lambda}{h^{-1}\mathrm{Mpc}}\right)^{-1}$$

where the limits are for $z = 1050$ in an Einstein-deSitter universe.

Sort anisotropies by three regions: large, medium, and small angular scales.

- $\theta > \theta_{hor} \approx 2^\circ$, $l < 100$

Dominated by Sachs-Wolfe effect of gravitational potential perturbations. Fairly model independent, insensitive to recombination details, some dependence on primordial spectrum tilt. Also probes recent ("secondary") large scale, e.g. supercluster, anisotropies.

- $\theta_{hor} > \theta > \theta_{str} \approx 10'$, $100 < l < 1000$

Dominated by velocity perturbations – Doppler rise, and density perturbations – photon-baryon coupling $\rightarrow$ acoustic oscillations. Sensitive to matter-radiation horizon size, baryon density. Reveals cosmological parameters.

- $\theta < \theta_{str} \approx 10'$, $l > 1000$

Coincidence of "structure" angles. Perturbation spectrum affected by Silk damping, last scattering surface thickness, inability to grow before matter-radiation equality.

Silk: $\lambda_S = 2.7\,(\Omega\Omega_b h^6)^{-1/4}$ Mpc

$$\theta_S \approx 4.5'\,\Omega^{3/4}, \qquad l_S \approx 760\,\Omega^{-3/4}$$

Last Scattering: $\lambda_{LS} = 7\,(\Omega h^2)^{-1/2}$ Mpc

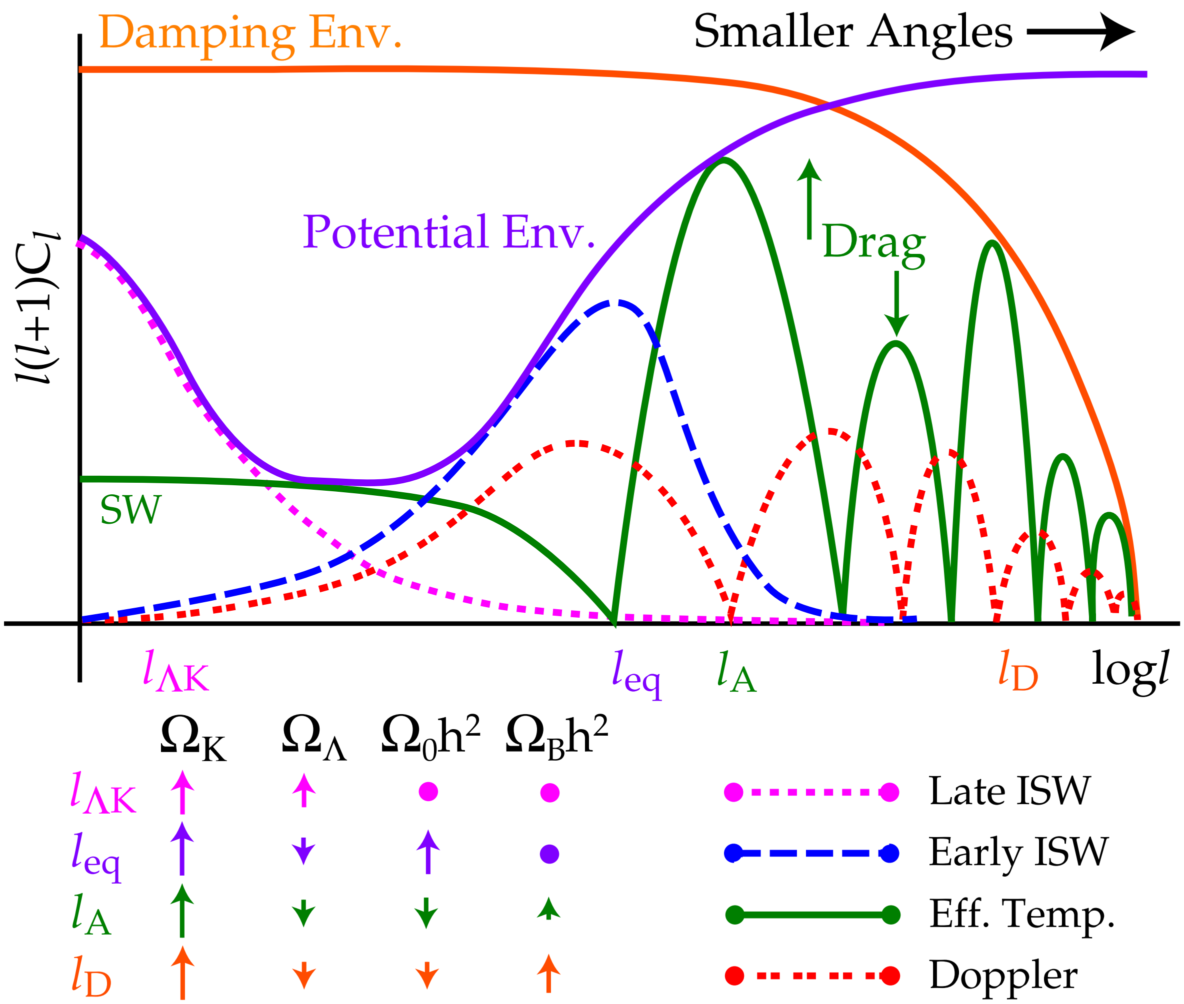
$$\theta_{LS} \approx 4'\,\Omega^{1/2}, \qquad l_{LS} \approx 860\,\Omega^{-1/2}$$

Equality: $\lambda_{eq} = 16\,(\Omega h^2)^{-1}$ Mpc

$$\theta_{eq} \approx 9'\,h^{-1}, \qquad l_{eq} \approx 380\,h$$

Also recent ("secondary") medium scale, e.g. cluster and galaxy, anisotropies such as Sunyaev-Zel'dovich and reionization.

**Measurements:**




*From http://www.sns.ias.edu/˜whu/physics/physics.html*

High precision differential measurements, 1 part in $10^5$ or better.

Dozens of groups, different techniques for different multipoles.

**Beam smearing** caused by finite detector resolution, temperature averaged over field. Gaussian beam with dispersion $\sigma$ gives **filter function**

$$W_l = e^{-l^2\sigma^2/2}$$

$$C_l \to C_l W_l^2$$

Other filter from **beam throw** – separation of fields, and geometry of observations.

Water absorbs microwaves so hard to observe through atmosphere.

Best with balloons, South Pole, or space.

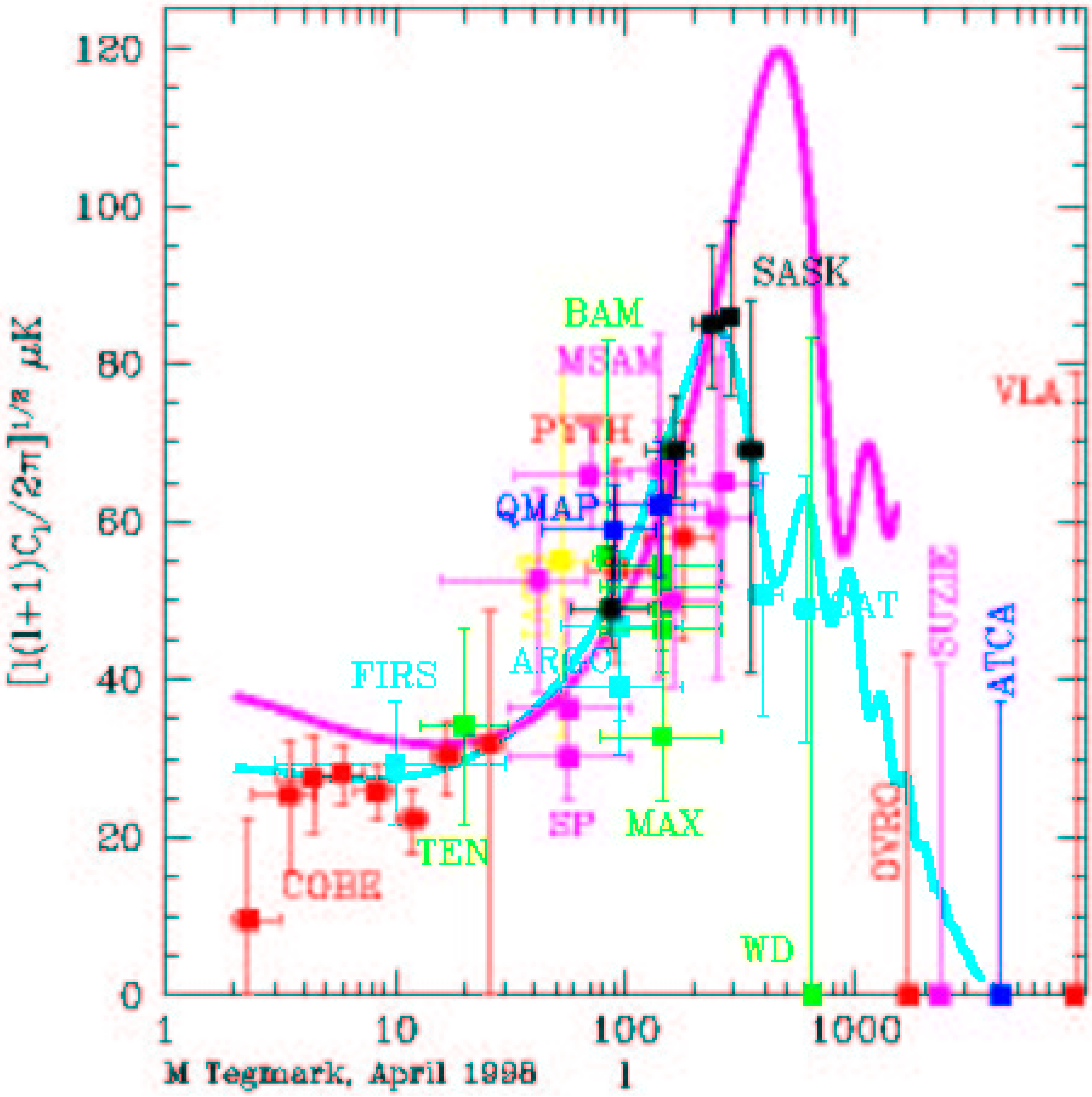
**COBE** (Cosmic Background Explorer) satellite observed $l = 2 - 20$ in 1992 through **DMR** (Differential Microwave Radiometer). Need large area of sky, good galaxy subtraction to get low multipoles.

Small angles, high multipoles give secondary anistropies. Ground based interferometers used.

Present focus on medium angles, acoustic peaks to determine cosmological parameters. Boomerang at UMass.

Future space missions:

- **MAP** (Microwave Anisotropy Probe, NASA 2001): $l = 2 - 1000$
- **Planck Surveyor** (ESA 2007): $l = 2 - 3000$

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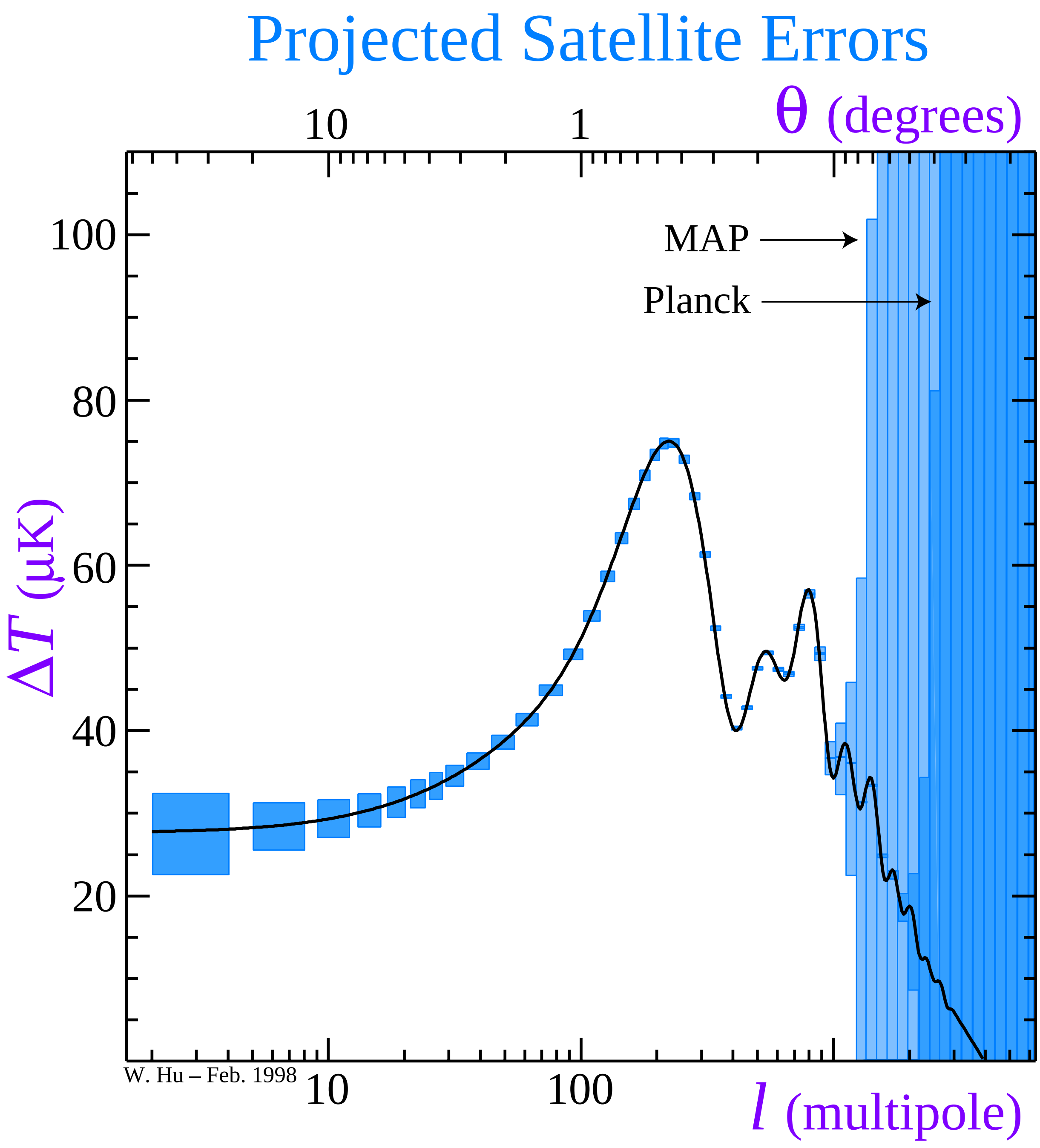


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