Simulation of the Effect of Errors on the SNAP's Measurement of the Cosmological Parameters

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I. Introduction

We have constructed simple toy models to determine how experimental errors propagate into the precision that SNAP can achieve in its measurement of the cosmological parameters. Specifically, we investigate the impact of errors on the proposed SNAP mission objectives – measurement of $\Delta\Omega_M/\Omega_M$ to $\leq 2\%$ and the measurement of $\Delta\Omega_\Lambda/\Omega_\Lambda$ to $\leq 5\%$. In this report, we consider three types of error: Gaussian distributed redshift errors, Gaussian distributed systematic errors, and non-Gaussian distributed systematic errors.

Gaussian distributed redshift errors are mostly the result of using photometry and low-resolution spectroscopy for the supernova redshift determination, rather than time-consuming, high-resolution spectroscopy. Gaussian distributed systematic errors and non-Gaussian distributed systematic errors are discussed in the document "Cosmological Parameters as a Function of z" also found in this Yellow Book. As described there, Gaussian distributed systematic errors introduce `an irreducible systematic error that is independent of other redshift bins'. The K-correction is an example of this type of systematic. Non-Gaussian distributed systematic errors, on the other hand, are those which are `correlated systematic magnitude shifts' that model errors introduced by supernova evolution, Malmquist bias, and gray dust.

II. Toy Models

A. Initial Cosmological Models

We analyzed the effect of errors on the cosmological models listed in Table 1.

Table 1. Input Cosmological Models

Model	Ω_M^0	Ω_Λ^0	w^0	
1	0.28	0.72	-0.7	flat
2	0.40	0.10	-0.7	mass dominated
3	0.28	0.30	-0.7	$\Omega_M \approx \Omega_\Lambda$
4	0.28	0.50	-0.7	Λ dominated

In model 1 we assumed a flat universe prior so that the errors on the cosmological parameters are smaller. The curvature is unconstrained on the other models.

B. Procedure

All our computations were based on Alex Kim's program SYSTEMATIC.PRO. However, the χ^2 minimization method was replaced by MINUIT, the well-known CERN minimization routine. In addition, the entire suite of routines was ported to the ROOT C++ framework to take advantage of the CERN histogramming, plotting, and analysis packages. We also used the routines in *Numerical Recipes* when required.

We investigated each error type individually. We then compared the results of superposing all the error types with our theoretical expectation.

1. Gaussian Redshift Errors

We first investigated how errors in redshift propagate into cosmological parameter errors in the absence of photometric measurement errors. The supernova distribution we used in this study was taken from Table 7.2 in the SNAP proposal. In particular, we chose 2,366 supernova redshifts z_i, by a Monte Carlo method that preserves the number of supernova in each redshift bin of the table (width $\Delta z = 0.1$) and which assumes a linear increase in the number of supernova in each bin. Current instrumental designs for SNAP are now being considered that will follow even more supernovae.

For each input cosmology, we first computed the distance moduli, $(m-M)_i = m_i(z_i, \Omega_M^0, \Omega_\Lambda^0, w^0)$, for the supernovae, where "0" indicates the initial model parameters. We then chose a normally distributed redshift error, δz_i, for each supernova using a constant standard deviation σ_z in the range s_z = 0.001-0.01, and used MINUIT to minimize the statistic

$$c^2 = \sum_{i=1}^{2,366} [\Delta M_B^0 + m_i(z_i + dz_i, \Omega_M, \Omega_\Lambda, w) - m_i(z_i, \Omega_M^0, \Omega_\Lambda^0, w^0) - as^0 \times dz_i/(1+z_i)]^2 / s_i^2$$

for the best-fitting cosmological parameters Ω_M, Ω_Λ, w, and ΔM_B^0. In this expression, $\Delta M_B^0 = M_B - M_B^0$ measures the best-fit SNIa absolute magnitude compared with the input absolute magnitude. For each supernova, the phenomenological stretch correction is given by the relation $Dm = a(s-1)$. Since Dm must be corrected for time dilation, the error in the stretch correction is written $\Delta s = as^0 \times dz/(1+z)$. In this computation, we used α = -1.74 and s^0 = 1. The factor s_i represents the photometric error and is constant since all supernovae will have the same quality observations. In our computations, MINUIT minimizes χ^2 with respect to four fit parameters, W_M, W_L, w, and ΔM_B^0, and we allowed these parameters to vary over the ranges shown in Table 2.

Table 2. Allowed Ranges for Fit Parameters in MINUIT

$0.1 \le \Omega_M \le 0.55$	$-1.0 \le \Omega_\Lambda \le 1.0$	$-1.0 \le w \le 0$	$-0.1 \le dM_B^0 \le 0.1$

We repeated this process for 200 trials for each input cosmology and made histograms of the quantities $\Delta\Omega_M = \langle\Omega_M\rangle - \Omega_M^0$, $\Delta\Omega_\Lambda = \langle\Omega_\Lambda\rangle - \Omega_\Lambda^0$, $\Delta w = \langle w\rangle - w^0$, and ΔM_B^0. We then increased the input standard deviation, s_z, and repeated the computations. Examples of these histograms for the four input cosmologies and $s_z = 0.003$ are shown in Figures 1a, 1b, 1c, and 1d.

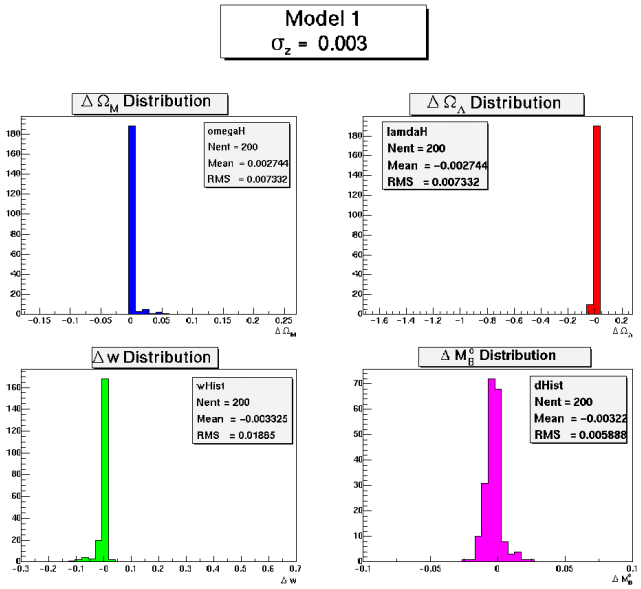

Figure 1a

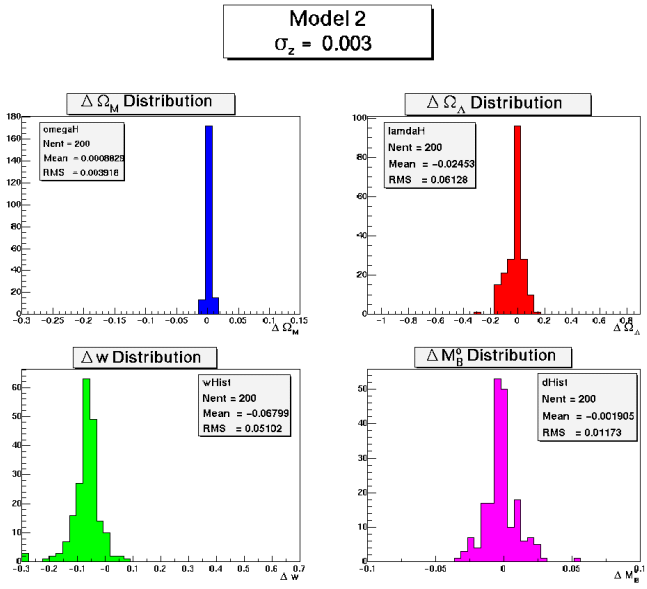

Figure 1b

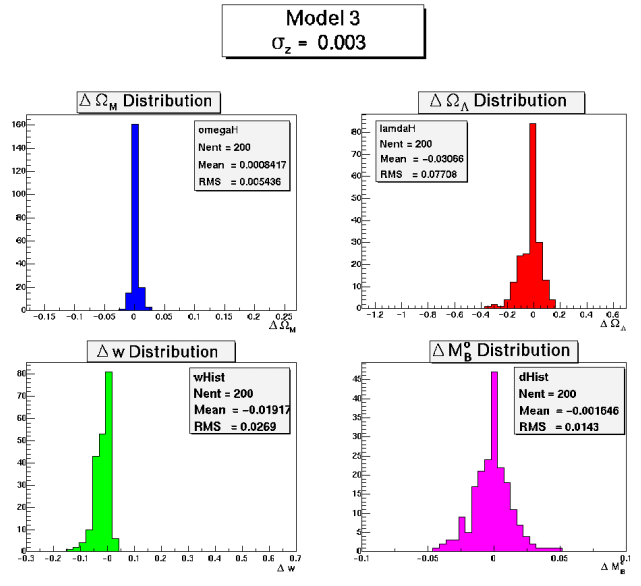

Figure 1c

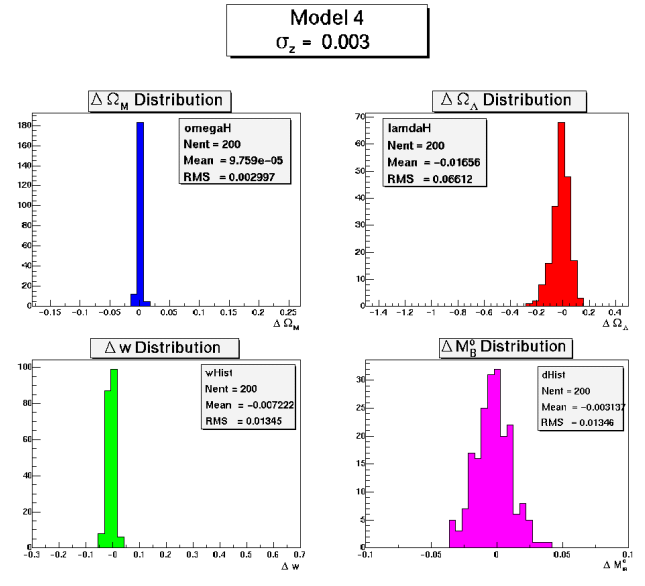

Figure 1d

2. Gaussian Systematic Errors

For these computations we used one supernova in each of 170 redshift bins of width Δz = 0.01, and we assumed that the redshifts of these supernovae were at the bin midpoint. For each initial cosmological model, we first chose the standard deviation for the photometric errors in each redshift bin, $s_m(z)$, according to the relation

$$s_m(z) = \Delta m_{\max} \times z / 1.7,$$

where Dm_{max} is in the range 0.005-0.035 magnitudes, and then determined a normally distributed error in the measured magnitude, Dm_i, for each supernova in the study. We used MINUIT to find the best-fitting cosmological parameters Ω_M, Ω_Λ, w, and ΔM_B^0 by minimizing the statistic

$$c^2 = \sum_{i=1}^{170} \left\{ \Delta M_B^0 + m_i(z_i, \Omega_M, \Omega_\Lambda, w) - \left[m_i(z_i, \Omega_M^0, \Omega_\Lambda^0, w^0) + \Delta m_i(z_i) \right] \right\}^2 / s_i^2 .$$

We varied the parameters over the same ranges as in the case of statistical errors. We repeated this process for 500 trials for each choice of cosmological model and mean magnitude error, and computed the relevant histograms. We repeated this process for 500 trials for each input cosmology and made histograms of the quantities $\Delta\Omega_M = \langle \Omega_M \rangle - \Omega_M^0$, $\Delta\Omega_\Lambda = \langle \Omega_\Lambda \rangle - \Omega_\Lambda^0$, $\Delta w = \langle w \rangle - w^0$, and ΔM_B^0. We then increased the standard deviation, $s_m(z_{max})$, and repeated the computations.

3. Non-Gaussian Systematic Errors

For these computations we used one supernova in each of 170 redshift bins of width Dz = 0.01. We assumed that the redshifts of the supernovae were at the bin midpoint.

For each initial cosmological model, we chose a systematic photometric offset in each redshift bin according to the relation

$$\langle \Delta m(z) \rangle = \Delta m_{\max} \times z / 1.7,$$

where $\Delta m_{\max}$ is in the range 0.005-0.035 magnitudes. In this case, the offset was applied directly to the measured magnitude for a single supernova at the center of each redshift bin. We used MINUIT to find the best-fitting cosmological parameters Ω_M, Ω_Λ, w, and ΔM_B^0. We varied the fit parameters over the same ranges as in the previous cases and minimized the same χ^2 as in the case of Gaussian systematic errors. Since the errors are always the same, there is no need to repeat the process.

III. Results

A. Gaussian Redshift Errors

Figures 2a, 2b, 2c, and 2d show the rms of the distributions for 200 trials as a function of the mean redshift error for the four input cosmologies. In Figures 3a, 3b, 3c, and 3d, we show the deviation from the mean as a function of the mean redshift error for the four models. The error bars in Figures 3 represent the errors in the mean. On Figures 2a-d, straight line fits to the rms as a function of mean redshift errors is superposed for range in which the fits are appropriate. On these figures the fit parameters are also shown. In the construction of these figures, significant outliers were excluded.

For the flat case 1, shown in Figures 2a and 3a, the rms increases linearly until $\sigma_z \approx 0.008$, and then flattens out. In this case, deviations from the mean are <1% and insignificant. For the matter dominated case 2, shown in Figures 2b and 3b, the rms increases linearly for $\Delta\Omega_M$ and Δw for all values of σ_z. On the other hand, for $\Delta\Omega_\Lambda$ and ΔM_B^0, the rms increases linearly until $\sigma_z \approx 0.008$, and then begins to deviate quite considerably from the linear increase for larger errors. As can be seen from Figure 3b, $\Delta\Omega_\Lambda$ and Δw deviate considerably from their initial values for larger errors. What appears to be happening in the mass dominated cosmologies with large redshift errors, the solutions for Ω_Λ that MINUIT finds are not well constrained. Basically MINUIT finds solutions that tend toward $\Omega_\Lambda \gg 0$ when the redshift errors are large. In addition, since Ω_Λ is small, w will not be well determined and MINUIT continually runs up against its $w = -1$. As shown in Figures 2c and 3c, this behavior is still in evidence for the $\Omega_M \gg \Omega_\Lambda$ but it is not quite so strong. For the Ω_Λ dominant case, shown in Figures 2d and 3d, the behavior is somewhat different. In this case, MINUIT finds solutions tending toward larger values of Ω_Λ as redshift errors become large. However, since Ω_Λ is dominant, Δw is not as large as in the previous cases.

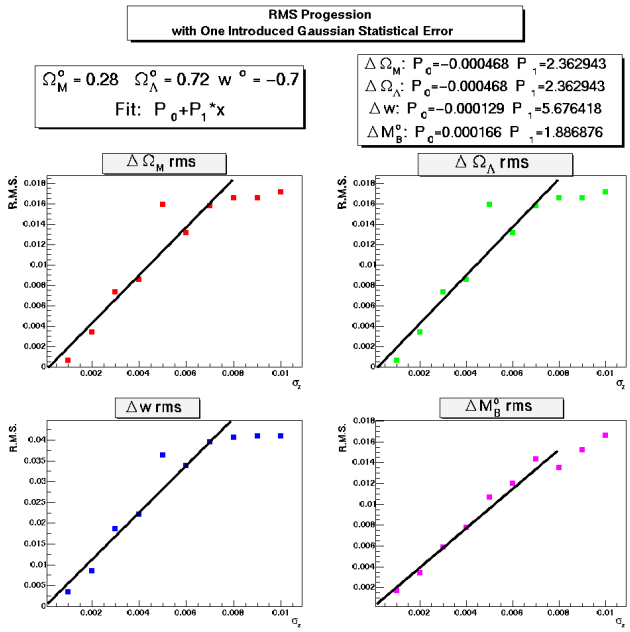

Figure 2a

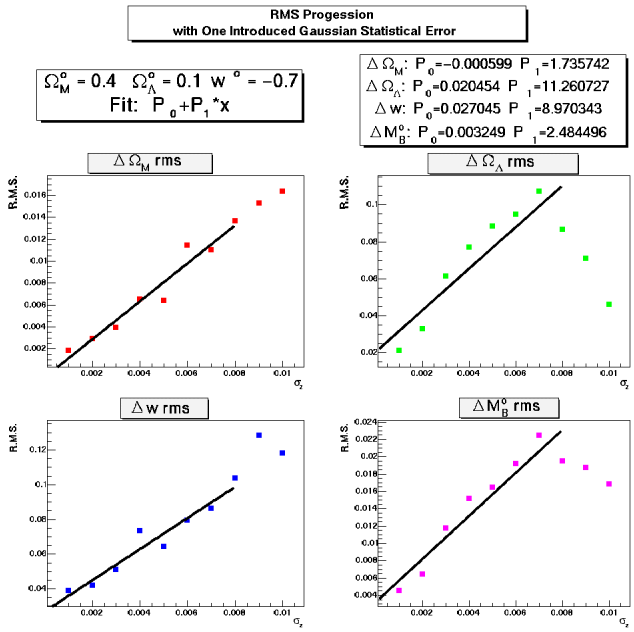

Figure 2b

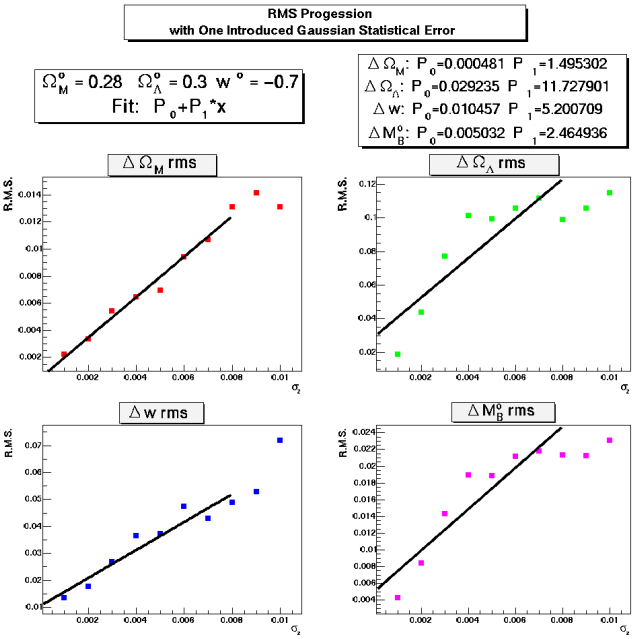

Figure 2c

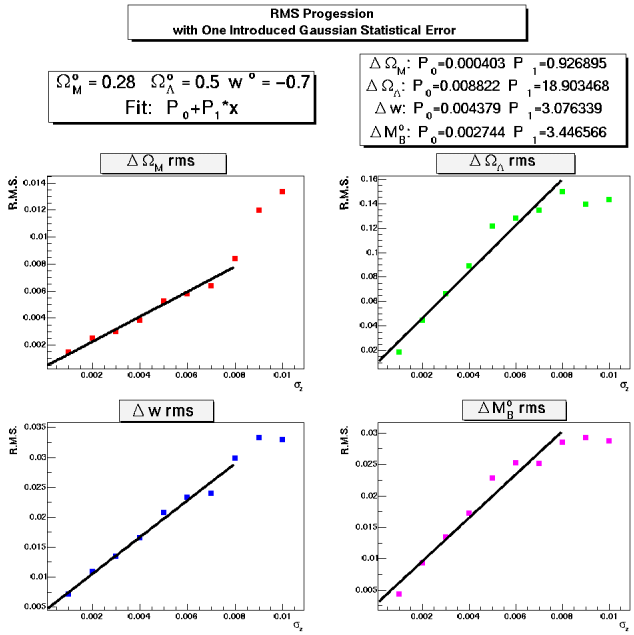

Figure 2d

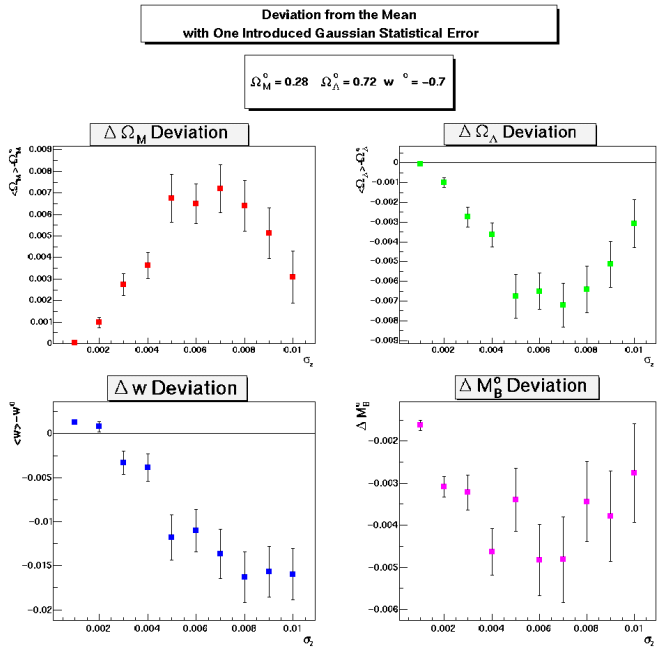

Figure 3a

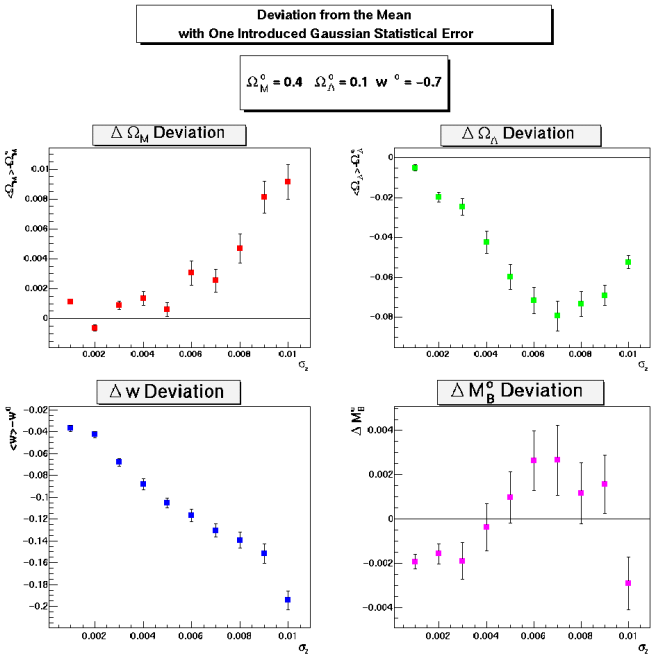

Figure 3b

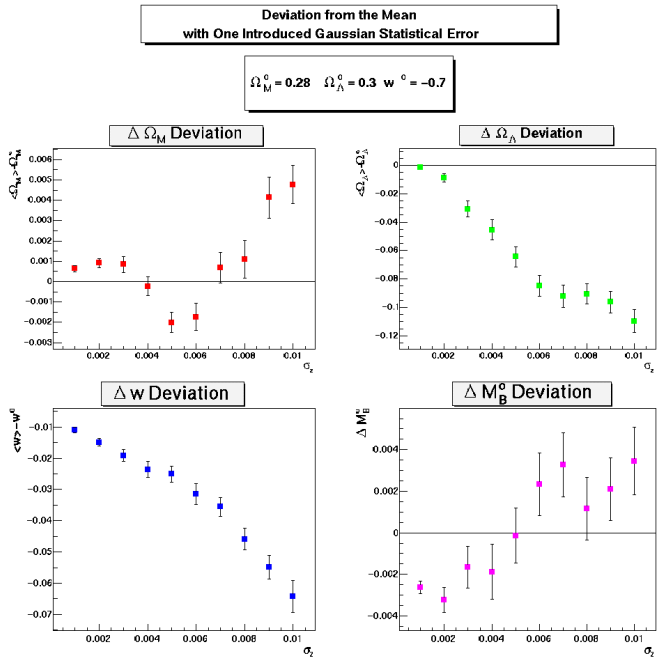

Figure 3c

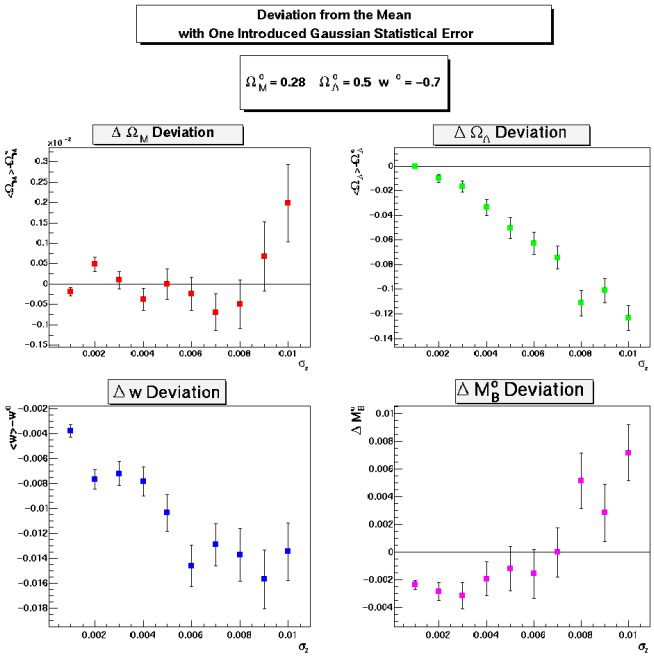

Figure 3d

B. Gaussian Systematic Errors

Figures 4a, 4b, 4c, and 4d show the r.m.s. of the distributions for 500 trials as a function of the magnitude of Dm_{max} for the four input cosmologies. The rms of the parameter distributions clearly increase linearly over the range of Dm_{max} studied. Except in one instance, the deviations from the mean of the cosmological parameters are < 0.01 for this error type; that is, within computational errors, $\langle \Omega_M \rangle - \Omega_M^0 \approx 0$, $\langle \Omega_\Lambda \rangle - \Omega_\Lambda^0 \approx 0$, $\langle w \rangle - w^0 \approx 0$, and $\langle \Delta M_B^0 \rangle \approx 0$. For model 2, $\langle w \rangle - w^0 \approx 0.05$ for $Dm_{max} = 0.035$.

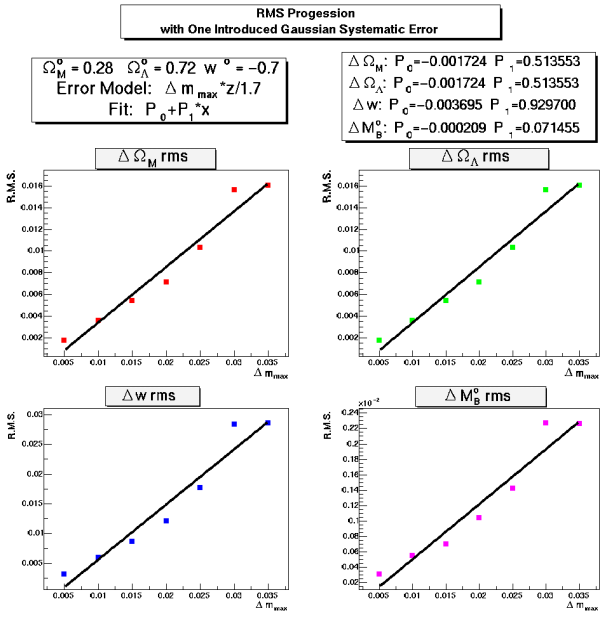

Figure 4a

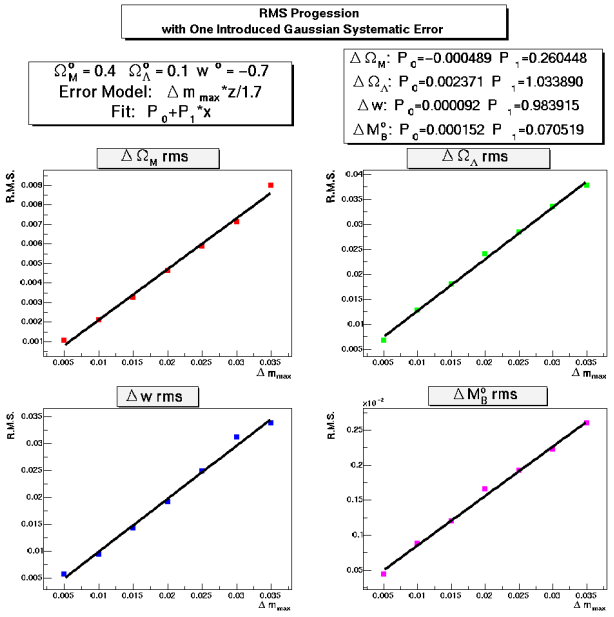

Figure 4b

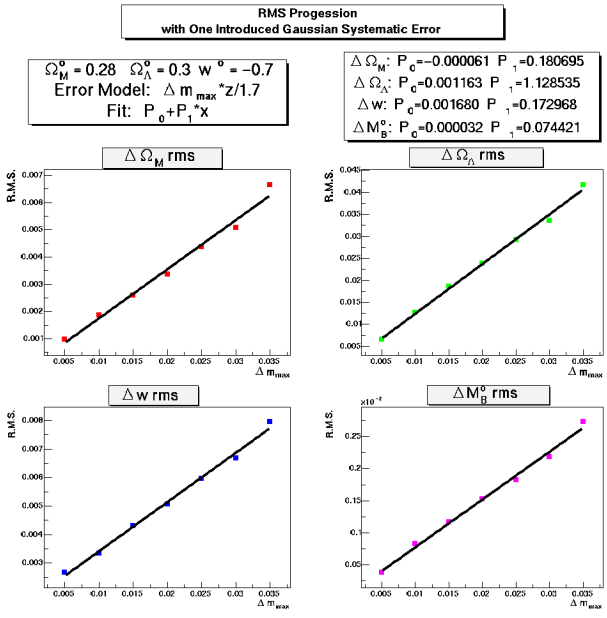

Figure 4c

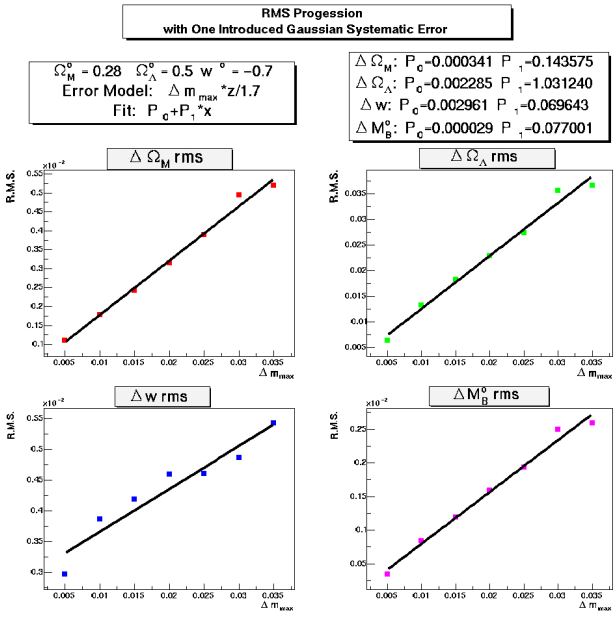

Figure 4d

C. Non-Gaussian Systematic Errors

Figures 5a, 5b, 5c, and 5d show the deviation from the mean of the cosmological parameters as a function of the magnitude of $\Delta m_{\max}$ for the four input cosmologies. The deviations in ΔM_B^0 are small and insignificant for all input cosmologies. For the flat case model 1, the deviations $\langle \Omega_M \rangle - \Omega_M^0$ increase with D_{max}; in addition, the behavior is highly variable for the largest values of Dm_{max}. For the remaining input cosmologies, $\langle \Omega_M \rangle - \Omega_M^0$ decreases with increasing Dm_{max}. Similar behavior is also seen for $\langle w \rangle - w^0$, except that $\langle w \rangle - w^0 \approx 0$ for small values of Dm_{max}. The deviations $\langle \Omega_\Lambda \rangle - \Omega_\Lambda^0$ decrease for all input cosmologies. In model 4 it appears as if MINUIT is making a transition to a different minimum in parameter space as Dm_{max} increases.

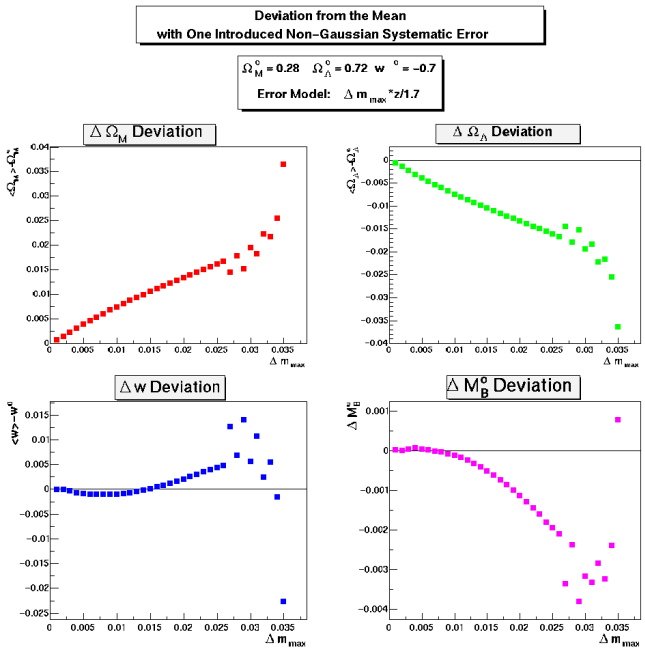

Figure 5a

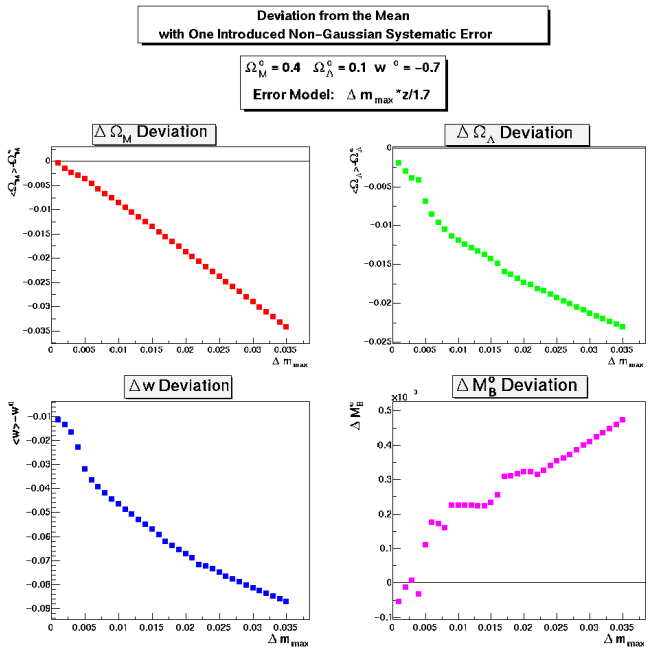

Figure 5b

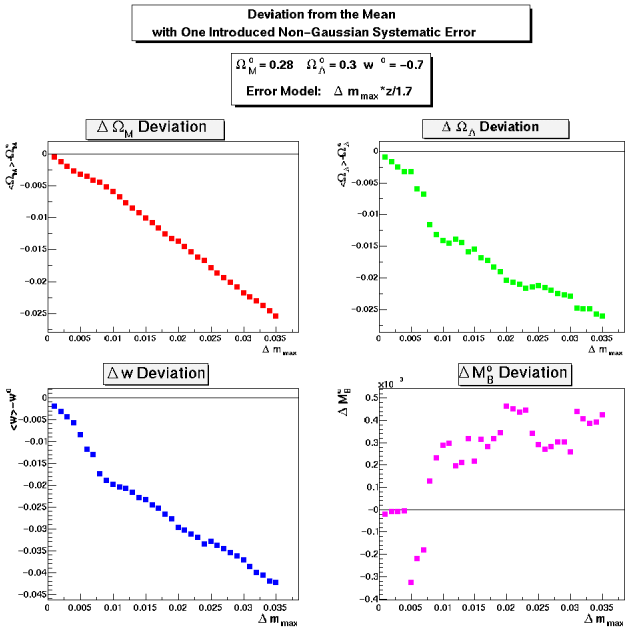

Figure 5c

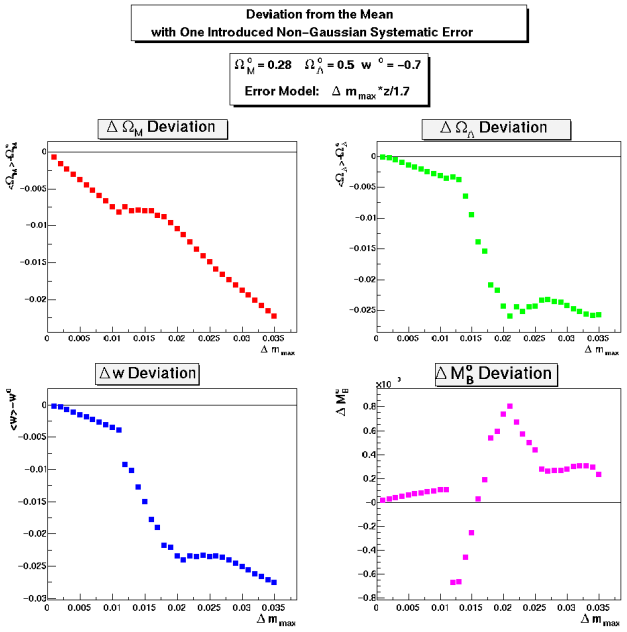

Figure 5d

D. Models with Multiple Error Types

Using the results above we can compute the combined effect of multiple error types on the cosmological parameters. We first determine the rms for the combined model by adding the rms of the Gaussian statistical errors and the Gaussian systematic errors in quadrature. We then compute the deviation from the mean for the cosmological parameters by simply adding the deviation from the mean for the non-Gaussian systematic errors.

We tested this hypothesis by running 200 trials each for a combined error case. For these computations we chose redshift errors as in the Gaussian statistical error case for the 2,366 SNIa. We chose a Gaussian systematic error using

$$s_m(z) = \Delta m_{\max} \times z/1.7,$$

and a non-Gaussian systematic error in the supernova magnitude using

$$\langle \Delta m(z) \rangle = \Delta m_{\max} \times z/1.7.$$

We then used MINUIT to find the best-fitting cosmological parameters by minimizing the statistic

$$c^2 = \sum_{i=1}^{2,366} [\Delta M_B^0 + m_i(z_i + dz_i, \Omega_M, \Omega_\Lambda, w) - m_i(z_i, \Omega_M^0, \Omega_\Lambda^0, w^0) - \Delta m_i(z_i) - as^0 \times dz_i / (1+z_i)]^2 / s_i^2$$

We show the results of the computations in Figures 6. We summarize the results in Table 3 below. For this case the predictions match the computations quite well.

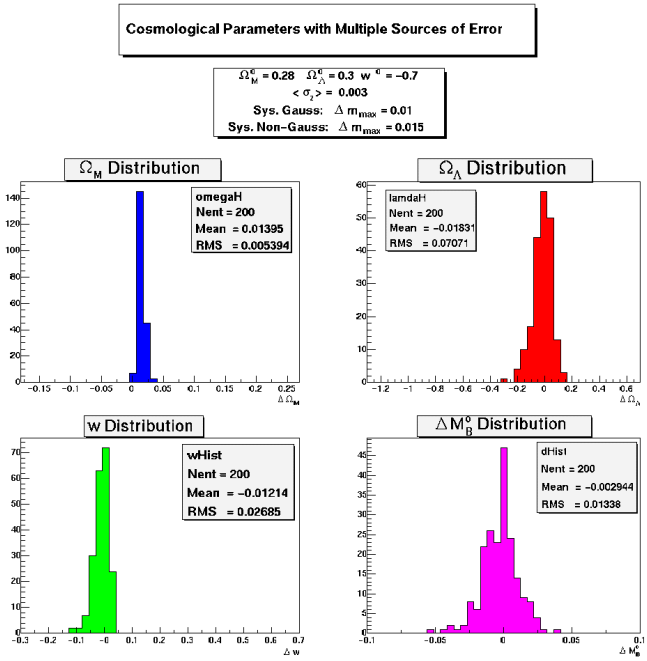

Figure 6

Table 3. Combined Error Model

Model	3
Gaussian Statistical	.003
Gaussian Systematic	.01
Non-Gaussian Statistical	.015
$s(\Omega_M)_{pred} - s(\Omega_M)_{comp}$	≈ 0
$[\langle\Omega_M\rangle - \Omega_M^0]_{pred} - [\langle\Omega_M\rangle - \Omega_M^0]_{comp}$	.001
$s(\Omega_\Lambda)_{pred} - s(\Omega_\Lambda)_{comp}$	.006
$[\langle\Omega_\Lambda\rangle - \Omega_\Lambda^0]_{pred} - [\langle\Omega_\Lambda\rangle - \Omega_\Lambda^0]_{comp}$	.001
$s(w)_{pred} - s(w)_{comp}$	.002
$[\langle w\rangle - w^0]_{pred} - [\langle w\rangle - w^0]_{comp}$	.01
$s(dM_B^0)_{pred} - s(dM_B^0)_{comp}$	≈ 0
$\langle dM_B^0\rangle_{pred} - \langle dM_B^0\rangle_{comp}$	≈ 0