Scalar-Tensor Gravity:

Constants Aren’t and Variables Don’t

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DarkFun 3 Workshop

Why Scalar-Tensor?

Scalar-tensor theories considered as alternative to GR since Jordan-Brans-Dicke. Cosmological use by Wagoner (1970).

ST are fundamental modifications of Einstein-Hilbert action, rather than phenomenological alterations of Friedmann equations or w(a).

Interest in curvature modifications -- high curvature for inflation and from string theories, low curvature for late acceleration and to solve coincidence problem. But f(R) theories equivalent to ST.

Extended Quintessence

Make the scalar field be useful -- drive late acceleration just as the inflaton drives inflation. Such ST theories are "extended quintessence".
Perrotta, Baccigalupi, Matarrese 2000

EH action $R/(16\pi G) \rightarrow F(\phi)R/2$

Scalar field equation of motion

$$\ddot{\phi} + 3H\dot{\phi} = -V_{\phi} + F_{\phi}R/2$$

Coupling to R drives scalar field evolution -- "R-boost" mechanism -- giving attractor, independent of V_{ϕ}. This solves fine tuning problem, even for $V=\Lambda$. Baccigalupi, Matarrese, Perrotta 2000

ST Expansion History

Cosmic expansion (Friedmann equation)

$$3FH^2 = \rho_{fluid} + V + \phi_t^2/2 - 3HF_\phi\phi_t$$

Brans-Dicke parameter is $\omega_{JBD} = F/F_\phi^2$.
Today, $F(\phi_{today}) = 1/(8\pi G)$.

On attractor, expect slow roll, so

$$\phi_t = F_\phi R/(6H) = H(1\text{-}q)\, F_\phi$$

Can define effective dark energy density

$$(3/8\pi G)\,\delta H^2 = V + (1/2)F_\phi^2H^2(q-1)(q+5) + 3H^2\,[F-1/(8\pi G)]$$

Effective Dynamics

Given effective dark energy (modified Friedmann), can define equation of state

$$w_0 = -1 + c_0/\omega_{JBD} \qquad\qquad w' = c_a/\omega_{JBD}$$

$$c_0 = [16-32\Omega_m+18\Omega_m^2-9\Omega_m^3]/[24\Omega_m] \approx 1.1$$

$$c_a = [\Omega_m/(1-\Omega_m)][24\Omega_m-45\Omega_m^2+18\Omega_m^3]/8 \approx 0.2$$

Baccigalupi, Garbari, Linder, Matarrese, Perrotta 2007

Solar system constraints imply within 3×10^{-5} of Λ. So although Newton's constant isn't constant, the dynamical variation doesn't vary!

ST Growth History

R not only drives ϕ, but ϕ drives R, i.e. affects matter density perturbations.

Three main effects: 1) generalized gravitational potentials Φ and Ψ ; 2) coupling $1/(8\pi G) \rightarrow F$; 3) nonzero anisotropic stress $\Pi = \Psi + \Phi = \delta F/F$.

Modified Poisson equation

$$2(k^2/a^2)\Phi = F^{-1}[\delta\rho_m + \Psi H^2 F_\phi{}^2(6(1-q)-(1-q)^2) - 3\dot{\Phi} H F_\phi{}^2(1-q)$$

$$+ \Pi \text{ terms}]$$

Expansion may look like Λ, but growth shows modGR.

Parameterized Beyond-Einstein Growth

We would like something similar to the PPN formalism to describe the accelerating universe.

Also want to cleanly separate expansion history and growth history to see the nature of the physics: physical dark energy vs. new gravity theory.

Expansion history described by $H(a)$ or $\Omega_m(a)$ or Ω_m, $w(a)$. Function → parameters, e.g. $w(a)=w_0+w_a(1-a)$.

But what about growth?

Growth Beyond Λ

Linear density perturbations in matter $\delta=\delta\rho_m/\rho_m$

$$\frac{d^2\delta}{dt^2}+2H(a)\frac{d\delta}{dt}-4\pi\rho_m\delta=0,$$

Normalize by matter dominated behavior
$G=d\ln(\delta/a)/d\ln a$

$$\frac{dG}{d\ln a}+\left(4+\frac{1}{2}\frac{d\ln H^2}{d\ln a}\right)G+G^2+3+\frac{1}{2}\frac{d\ln H^2}{d\ln a}-\frac{3}{2}\Omega_m(a)=0,$$

Exact solution

Linder & Cahn
astro-ph/0701317

$$G(a)=-1+[a^4H(a)]^{-1}\int_0^a\frac{da'}{a'}\,a'^4H(a')\left[1+\frac{3}{2}\Omega_m(a')-G^2(a')\right].$$

When matter dominated, $G\to 0$. Even today $G\approx -1/2$.
So take $G^2 \ll 1$ for explicit solution (good in δ to 0.2%).

Growth Beyond Λ

Growth as function of dark energy

$$G(a) = -\frac{1}{2}\Omega_w(a) - \frac{1}{4}a^{-5/2}\int_0^a \frac{da'}{a'}\, a'^{5/2}\Omega_w(a').$$

Now recall gravitational growth index γ

Linder 2005
PRD 72, 043529

$$G(a) = \Omega_m(a)^\gamma - 1.$$

Growth index determined as

$$\gamma \;\approx\; \frac{1}{2} + \frac{1}{4}\int_0^1 \frac{du}{u}\, u^{5/2}\Omega_w(au)/\Omega_w(a).$$

E.g. dark energy $\Omega_w(a \ll 1) \sim a^{-3w}$

$$\gamma_\infty = \frac{3(1-w_\infty)}{5-6w_\infty} \approx \frac{6}{11} + \frac{3}{121}(1+w_\infty).$$

Agrees excellently with Linder 2005 numerical fits.

Growth Beyond Λ

Gravitational growth index γ is nearly constant, i.e. **single parameter** (not function) to describe growth *separately* from expansion effects.

Simplest form that gives correct asymptotic limits of growth in past and future, plus accurate to 0.2% for growth to today.

Also describes early dark energy (%-level density contribution at CMB last scattering) and time varying dark energy, $w(a)$ (not just w_a).

But what about beyond-Einstein gravity?

Growth Beyond Einstein

Take source term $G_N\Omega_m(a)$ with varying gravity coupling: $G_N \rightarrow G_N\,(1+[Q(a)-1])$

$$\gamma \approx \frac{1}{2} + \frac{1}{4}\int_0^1 \frac{du}{u}\, u^{5/2}\Omega_w(au)/\Omega_w(a) - \frac{3}{2}\int_0^1 \frac{du}{u}\, u^{5/2}\,[Q(au)-1]/\Omega_w(a).$$

For $[Q(a)-1]/\Omega_w(a) \sim a^{3q+w}$, then q>-3w means gravity variation negligible, q<-3w gives large GR violation, so concentrate on scaling solution q=-3w -- motivated by physical relation between coupling and expansion. For $A = [Q(a)-1]/\Omega_w(a)$,

$$\gamma_\infty = \frac{3(1-w_\infty-A)}{5-6w_\infty}.$$

DGP Braneworld Gravity

For DGP braneworld gravity

Lue, Scoccimarro, Starkman 2004

$$(Q-1)_{\rm DGP} = -\frac{1}{3}\left(\frac{1-\Omega_m^2(a)}{1+\Omega_m^2(a)}\right).$$

So

$$A \equiv \frac{Q-1}{1-\Omega_m(a)} = -\frac{1}{3}\,\frac{1+\Omega_m(a)}{1+\Omega_m^2(a)} \to -\frac{1}{3},$$

$$\gamma_{\infty,\rm DGP} = \frac{11}{16} = 0.6875$$

Exact agreement with asymptotic numerical solution, excellent agreement with full growth history using Linder (2005) numerical fit γ=0.68.

Derived analytic fit $$\gamma_{\rm DGP} \approx \frac{7+5\Omega_m(a)+7\Omega_m^2(a)+3\Omega_m^3(a)}{[1+\Omega_m^2(a)]\,[11+5\Omega_m(a)]}$$

Relation to PPN

Growth index γ works for extended quintessence (scalar-tensor gravity) also.

PPN: $$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi)d\vec{x}^2$$

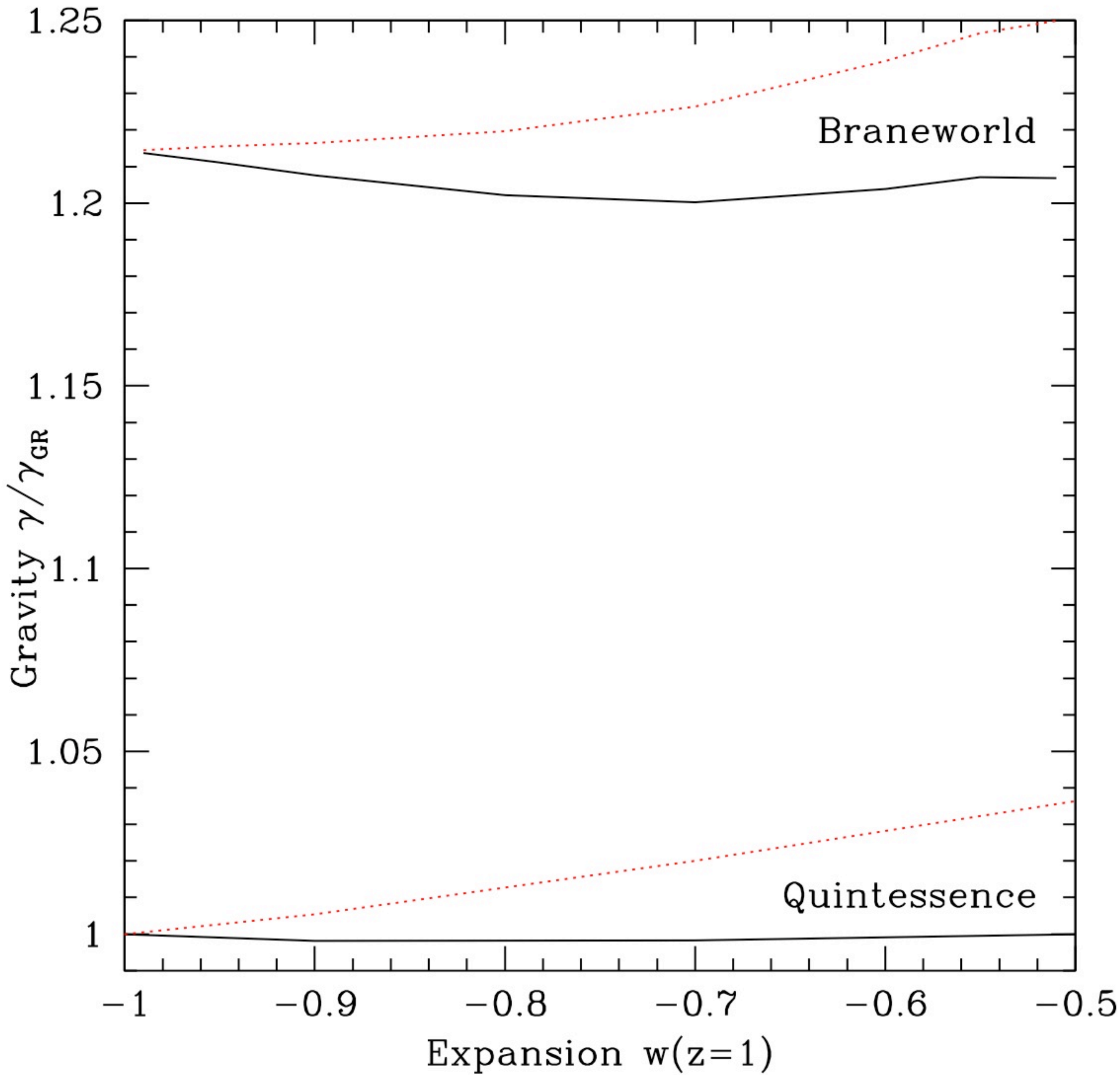
Main correction from anisotropic stress; Poisson equation takes

$$-k^2\Psi \to -k^2\Psi + (2/3)\,k^4(\Psi+\Phi)/\rho_m\,.$$

Coupling deviation (Q-1) ~ 1+ Φ/Ψ = 1 - γ_{PPN}

$$\gamma = \frac{3(1-w_\infty+(2/9)(k/H)^2[1-\gamma_{\rm PPN}])}{5-6w_\infty}.$$

Revealing the Nature of the Physics



Revealing the Nature of the Physics

Gravitational growth index γ is 1) nearly constant, i.e. single parameter (not function) to describe beyond-Einstein growth, 2) independent of expansion history, *separating* growth from expansion, 3) clear signal of beyond-Einstein physics: BW has 20% deviation from GR prediction, while "noise" from expansion *w* within GR is 0.2%.

Minimal Modified Gravity uses simultaneous fit to expansion and growth Huterer & Linder 2006, through $\{\Omega_m, w_0, w_a, \gamma\}$, as a benchmark model to explore the accelerating universe (cf. mSUGRA for dark matter).