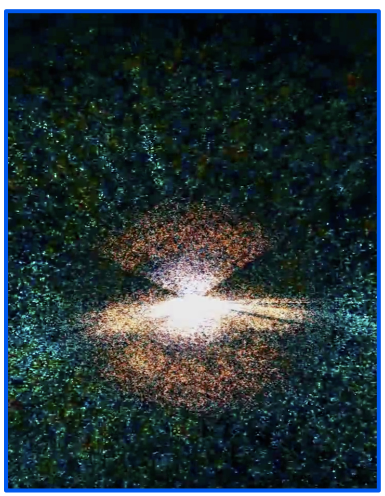
Physics of Cosmic Acceleration

2. Dark Energy as a Field

Eric Linder

II Tiomno School (Rio 2012)

UC Berkeley & Berkeley Lab
Institute for the Early Universe, Korea



What’s the Matter with Energy?

Why not just bring back the cosmological constant (Λ)?

When physicists calculate how big Λ should be, they don’t quite get it right.

They are off by a factor of

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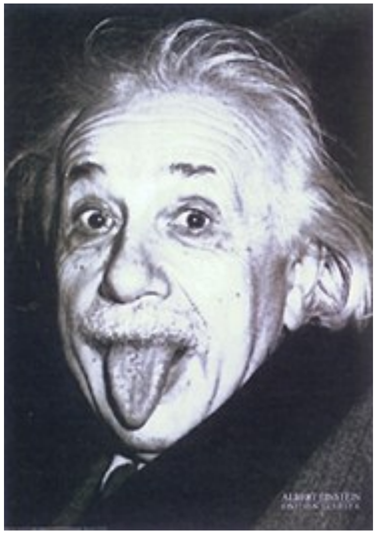


This is modestly called the fine tuning problem.

Cosmic Coincidence

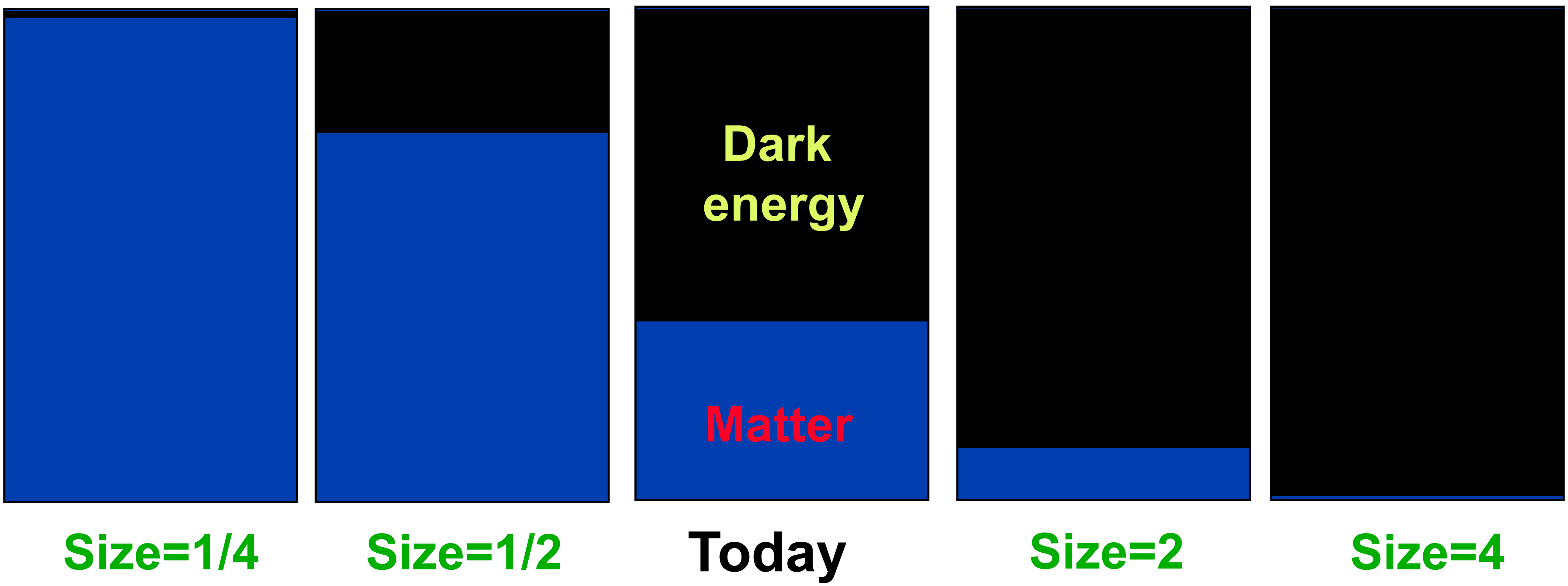
Why not just settle for a cosmological constant Λ?

→ For 90 years we have tried to understand why Λ is at least 10^{120} times smaller than we would expect – *and failed*.

→ We know there *was* an epoch of time varying vacuum once – inflation.

Cosmic Coincidence

We cannot calculate the vacuum energy to within 10^{120}. But it gets worse: Think of the energy in Λ as the level of the quantum "sea". At most times in history, matter is either drowned or dry.



Λ: Ugly Duckling

Astrophysicist:

Einstein equations –

Λg_{ab}

$\rightarrow \boxed{p = -\rho}$

Naturally, ρ =const= ρ_{PL}

$\boxed{\Omega_\Lambda = 10^{120}}$

Today $\Omega_\Lambda \approx \Omega_M$

Field Theorist:

Vacuum – Lorentz invariant

$T_{ab} \sim \eta_{ab}$ = diag { -1, 1, 1, 1}

$\rightarrow \boxed{p = -\rho}$

Naturally, $E_{vac} \sim 10^{19}$ GeV

$\mathcal{E}_\Lambda \sim (\mathrm{meV})^4$

$\boxed{\Lambda = 0?}$

- Fine Tuning Puzzle – why so small?
- Coincidence Puzzle – why now?

Theory of Fields

Scalar field:

At every point in a field of grass, you can measure the height of the grass: a single number or scalar h(x).

Vector fields:

At every point in a trampled field of grass, you can measure the length of the grass and the direction it is lying: a vector $\vec{g}(x)$.

On Beyond Λ!

"You'll be sort of surprised what there is to be found
Once you go beyond Λ and start poking around."
– Dr. Seuss, à la "On Beyond Zebra"

New quantum physics? Does nothing weigh something?

Einstein's cosmological constant, Quintessence, String theory

New gravitational physics? Is nowhere somewhere?

Quantum gravity, extended gravity, extra dimensions?

***We need to explore further frontiers* in high energy physics, gravitation, and cosmology.**

Finding Our Way in the Dark

Dark energy is a completely unknown animal.

A new theory or a new component?

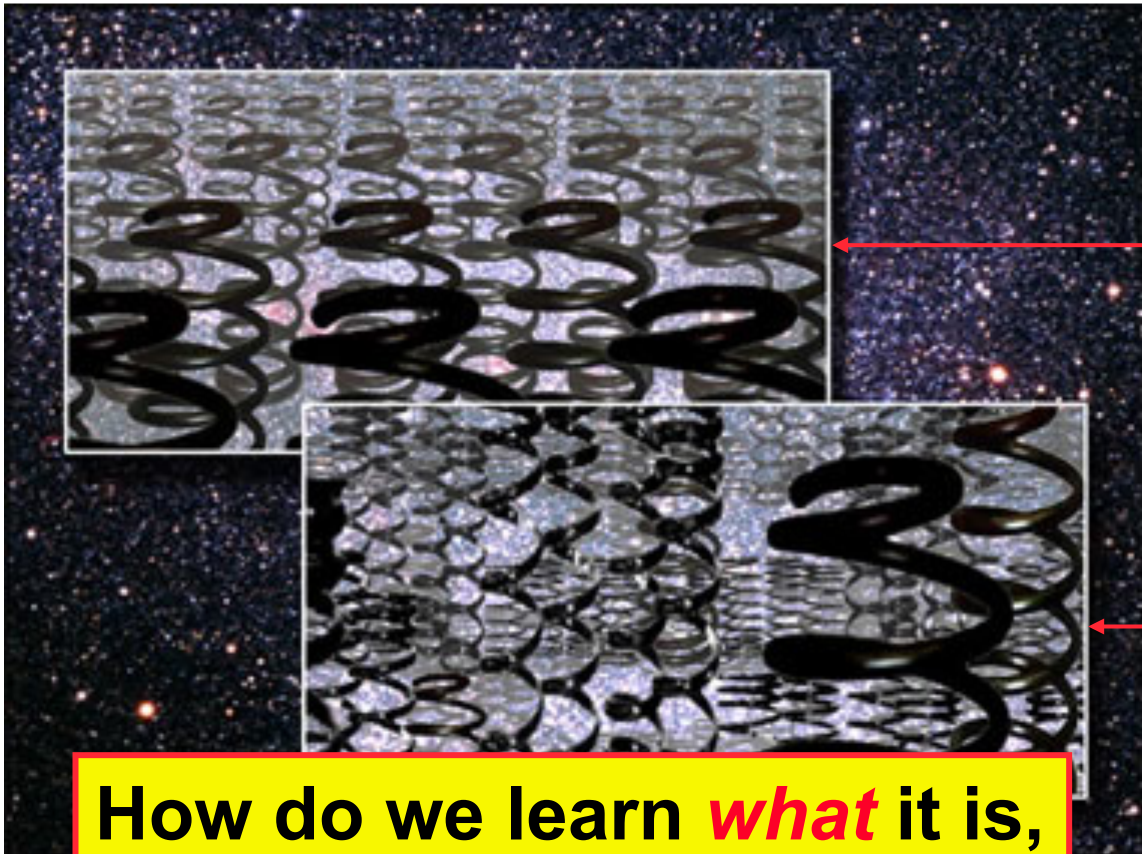
Track record:

Inner solar system motions → General Relativity

Outer solar system motions → Neptune

Galaxy rotation curves → Dark Matter

Nature of Acceleration



Is dark energy static? Einstein's cosmological constant Λ.

Is dark energy dynamic? A new, time- and space-varying field.

Is dark energy a change in **gravity**?

How do we learn *what* it is, not just *that* it is?

How much dark energy is there? Ω_{DE}

How springy/stretchy is it? $w=P/\rho$

A new law of gravity, or a new component? $G_N(k,z)$

Scalar Field Theory

Scalar field Lagrangian -
canonical, minimally coupled

$$\mathcal{L}_\phi = (1/2)(\partial_\mu \phi)^2 - V(\phi)$$

Noether prescription → Energy-momentum tensor

$$T_{\mu\nu} = (2/\sqrt{-g})\ [\ \delta(\sqrt{-g}\,\mathcal{L})/\delta g_{\mu\nu}\ ]$$

Perfect fluid form (from RW metric)

Energy density $\rho_\phi = (1/2)\,\dot{\phi}^2 + V(\phi) + (1/2)(\nabla\phi)^2$

Pressure $p_\phi = (1/2)\,\dot{\phi}^2 - V(\phi) - (1/6)(\nabla\phi)^2$

$+ (1/2)(\nabla\phi)^2$

Scalar Field Equation of State

Equation of state ratio

$$w = p/\rho$$

Klein-Gordon equation (Lagrange equation of motion)

$$\ddot{\phi} + 3H\dot{\phi} = -dV(\phi)/d\phi$$

Continuity equation follows KG equation

$$[(1/2)\dot{\phi}^2]^{\cdot} + 6H\,[(1/2)\dot{\phi}^2] = -\dot{V}$$

$$\dot{\rho} - \dot{V} + 3H\,(\rho+p) = -\dot{V}$$

$$d\rho/d\mathit{ln}\,a = -3(\rho+p) = -3\rho\,(1+w)$$

$$\rho_i(a) = \rho_i\, e^{-3\int_0^{\ln a} d\ln a'\,[1+w_i(a')]} \sim a^{-3(1+w_i)}$$

Equation of State

Limits of (canonical) Equations of State:

$$w = (K-V) / (K+V)$$

Potential energy dominates (slow roll)

$$V \gg K \Rightarrow w = -1$$

Kinetic energy dominates (fast roll)

$$K \gg V \Rightarrow w = +1$$

Oscillation about potential minimum (or coherent field, e.g. axion)

$$\langle V \rangle = \langle K \rangle \Rightarrow w = 0$$

Equation of State

Examples of (canonical) Equations of State:

$$d\rho/d\ln a = -3(\rho+p) = -3\rho\,(1+w)$$

ρ = (Energy per particle)(Number of particles) / Volume
$= E\,N\,a^{-3}$

Constant w implies $\rho \sim a^{-3(1+w)}$

Matter: $E\sim m\sim a^0$, $N\sim a^0 \rightarrow w=0$

Radiation: $E\sim 1/\lambda\sim a^{-1}$, $N\sim a^0 \rightarrow w=1/3$

Curvature energy: $E\sim 1/R^2\sim a^{-2}$, $N\sim a^0 \rightarrow w=-1/3$

Cosmological constant: $E\sim V$, $N\sim a^0 \rightarrow w=-1$

Anisotropic shear: $w=+1$
Cosmic String network: $w=-1/3$; Domain walls: $w=-2/3$

Dark Energy Models

Scalar fields can roll:

1) fast – “kination” [Tracking models]

2) slow – acceleration [Quintessence]

3) steadily – acceleration deceleration
[Linear potential]

4) oscillate – potential minimum [V~ ϕ^n],
pseudoscalar, PNGB (Frieman, Hill, Stebbins, Waga 1995)

Equation of State

Reconstruction from EOS:

$$\rho(a) = \Omega_\phi \rho_c \exp\{ 3 \int d\,ln\, a\, [1+w(z)] \}$$

$$\phi(a) = \int d\,ln\, a\, H^{-1} \,\mathrm{sqrt}\{ \rho(a)\, [1+w(z)] \}$$

$$V(a) = (1/2)\, \rho(a)\, [1-w(z)]$$

$$K(a) = (1/2)\dot{\phi}^2 = (1/2)\, \rho(a)\, [1+w(z)]$$

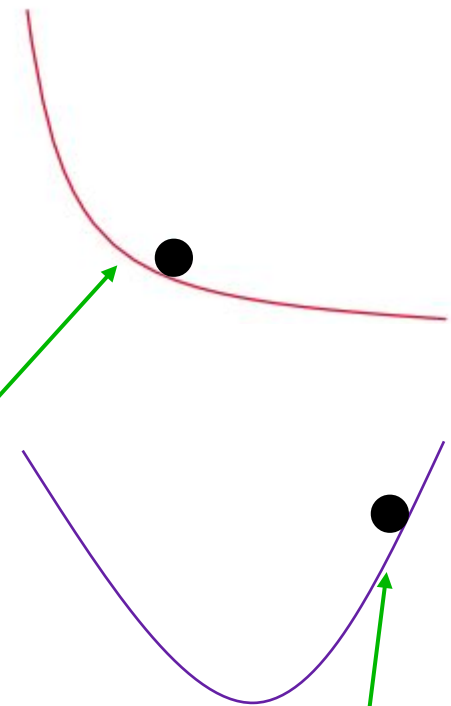
But, $\dot{\phi} \sim \sqrt{[(1+w)\rho]} \sim \sqrt{(1+w)}\, HM_p$

So if $1+w \ll 1$, then $\Delta\phi \sim \dot{\phi}/H \ll M_p$.

It is very hard to directly reconstruct the potential.

Goldilocks problem: Dark energy is unlike Inflation!

Dynamics of Quintessence



Equation of motion of scalar field

$$\ddot{\phi} + 3H\dot{\phi} = -dV(\phi)/d\phi$$

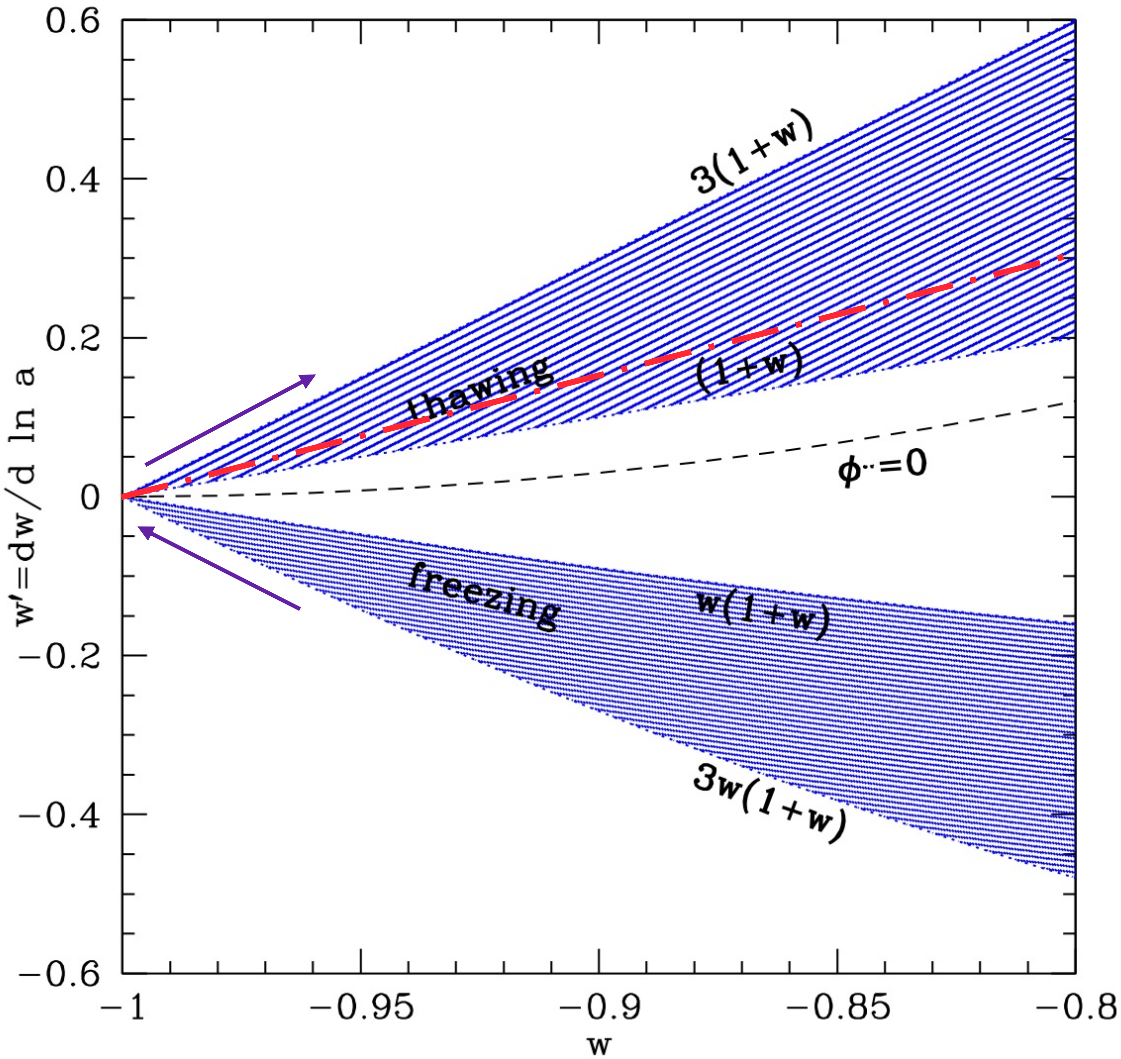
- driven by steepness of potential
- slowed by Hubble friction

Broad categorization – which term dominates:

- field rolls but decelerates as dominates energy
- field starts frozen by Hubble drag and then rolls

Freezers vs. Thawers

Limits of Quintessence



$$w = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)}$$

Distinct, narrow regions of w-w′

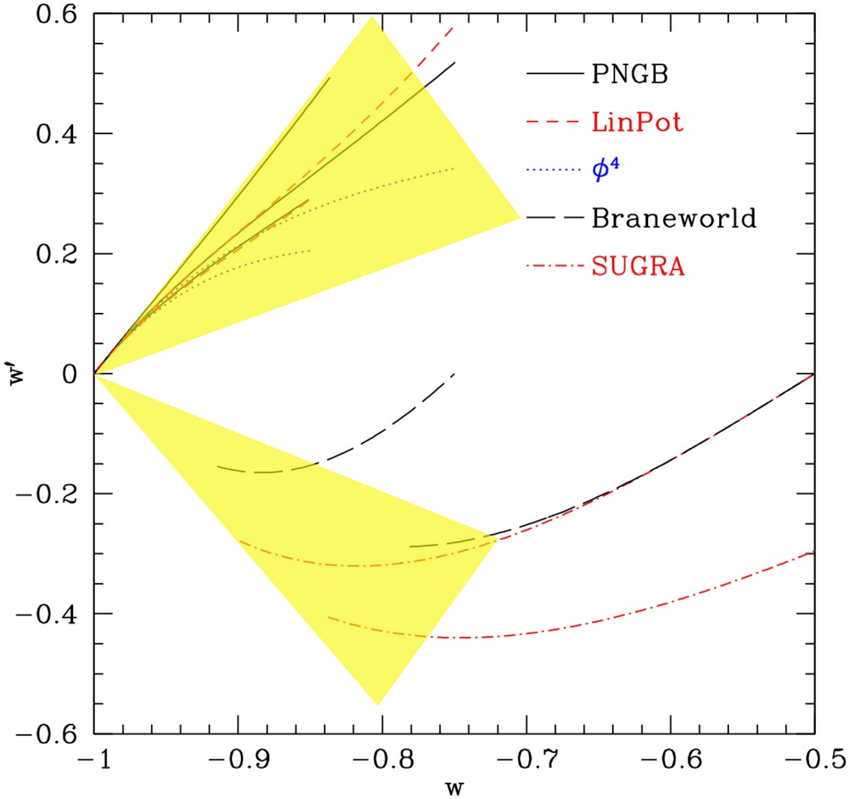
Caldwell & Linder 2005
PRL 95, 141301

Entire "thawing" region looks like <w> = -1 ± 0.05.

Need w′ experiments with σ (w′) ≈ 2(1+w).

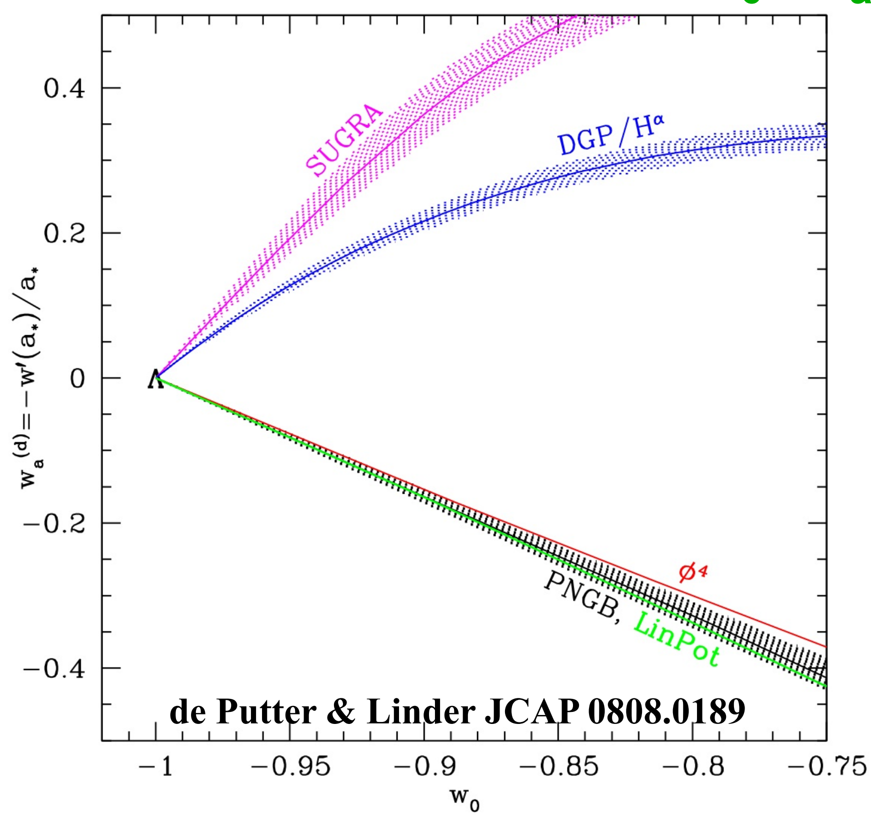
Calibrating Dark Energy

Models have a diversity of behavior, within thawing and freezing.



But we can calibrate w' by "stretching" it: $w' \to w'(a_*)/a_*$.

Calibrated parameters w_0, w_a.



The two parameters w_0, w_a achieve 10^{-3} level accuracy on observables d(z), H(z).

$$\boxed{w(a)=w_0+w_a(1-a)}$$

This is from physics (Linder 2003). It has *nothing* to do with a Taylor expansion.

Solving the Equation of Motion

Klein-Gordon equation $$\ddot{\phi} = -3H\dot{\phi} - \frac{dV}{d\phi}$$

Transform to new variables $$x \equiv \frac{\kappa\dot{\phi}}{\sqrt{6}\,H} \quad ; \quad y \equiv \frac{\kappa\sqrt{V}}{\sqrt{3}\,H} \qquad ' = \frac{d}{d\ln a}$$

$$H^2 = (\kappa^2/3)[\rho_m + (1/2)(\dot{\phi})^2 + V]$$

Autonomous system

$$x' = -3x + \lambda\sqrt{\frac{3}{2}}y^2 + \frac{3}{2}x\left[2x^2 + \gamma\left(1 - x^2 - y^2\right)\right]$$

$$y' = -\lambda\sqrt{\frac{3}{2}}xy + \frac{3}{2}y\left[2x^2 + \gamma\left(1 - x^2 - y^2\right)\right] \ ,$$

Copeland, Liddle, Wands 1998
Phys. Rev. D 57, 4686

where $$\kappa^2 = 8\pi G \quad ; \quad \gamma = 1 + w_b \quad ; \quad \lambda = \frac{-V_{,\phi}}{\kappa V}$$

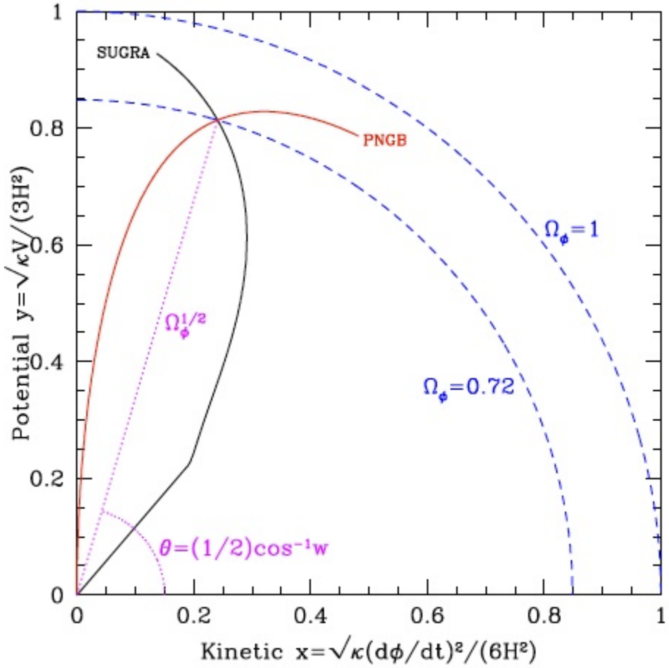
Transform solution to $$\Omega_\phi = x^2 + y^2 \quad ; \quad w = \frac{x^2 - y^2}{x^2 + y^2}$$

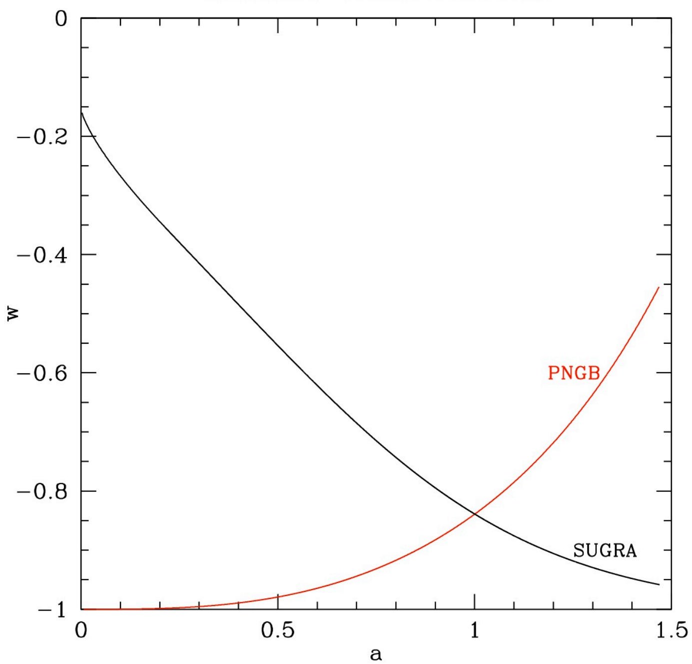
Can add equation for EOS dynamics

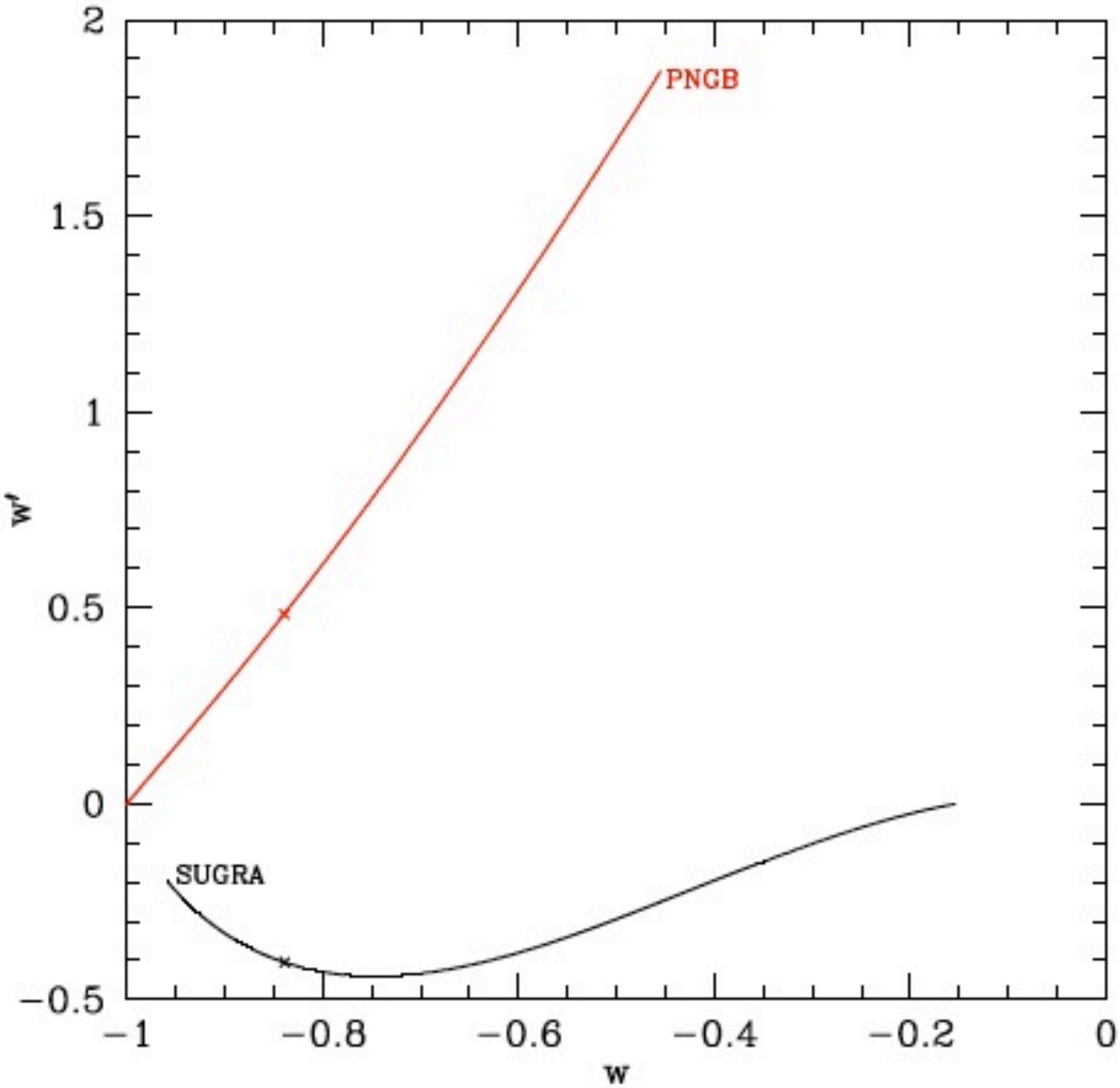
$$w' = -3(1 - w^2) + \lambda(1 - w)\sqrt{3(1 + w)\Omega_\phi}$$

Caldwell & Linder 2005
Phys. Rev. Lett 95, 141301

Equation of State Dynamics



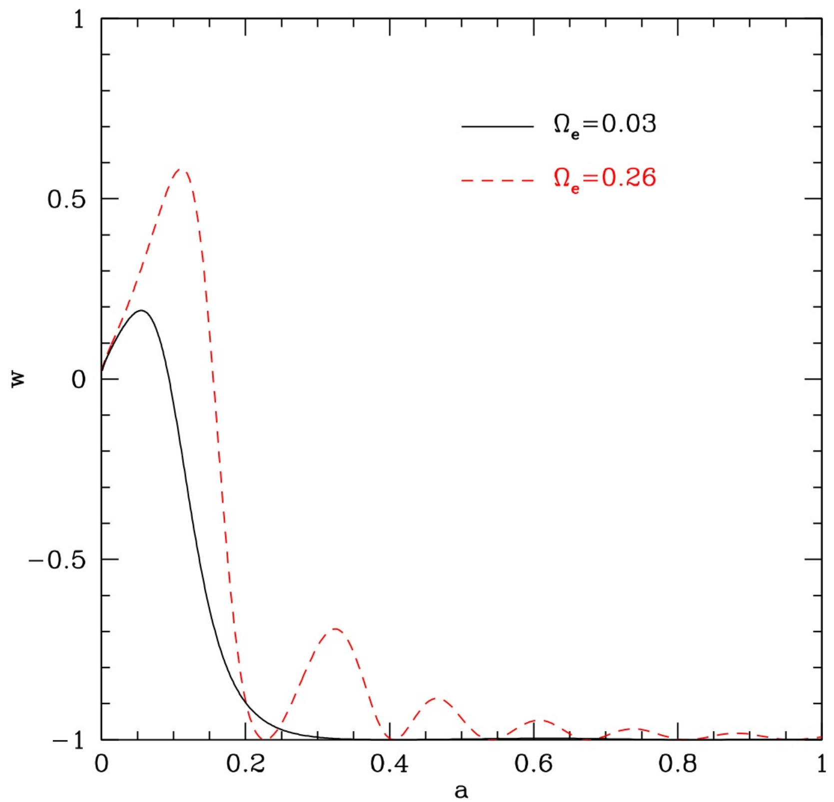


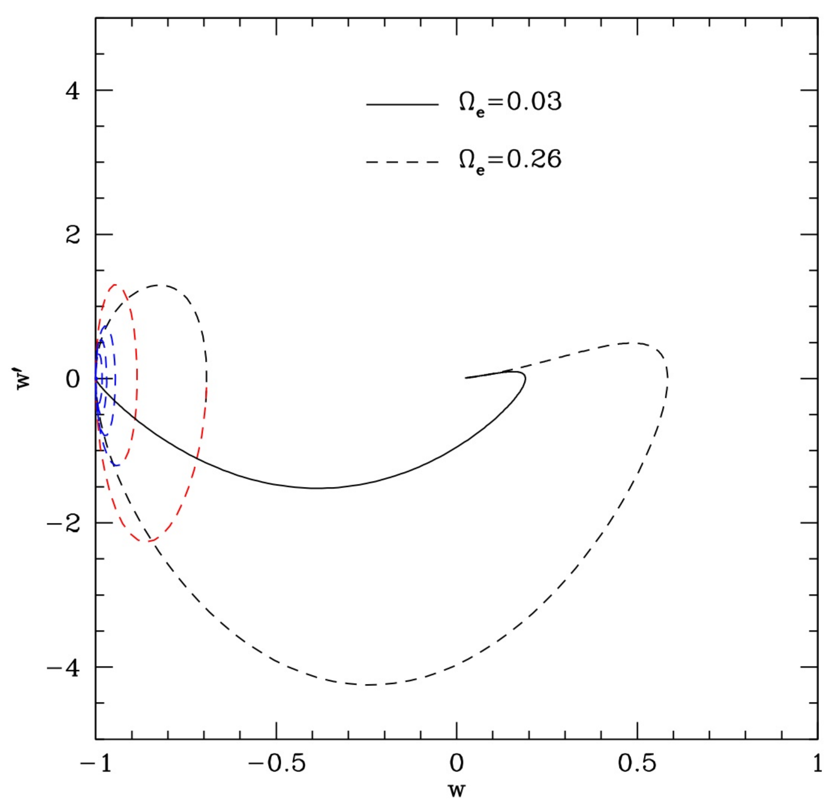


Equation of State Dynamics

For robust solutions, pay attention to initial conditions, shoot forward in time, use 4th order Runge-Kutta.

For monotonic Ω_ϕ, can switch to Ω_ϕ as time variable, defining present as, e.g. Ω_ϕ=0.72.



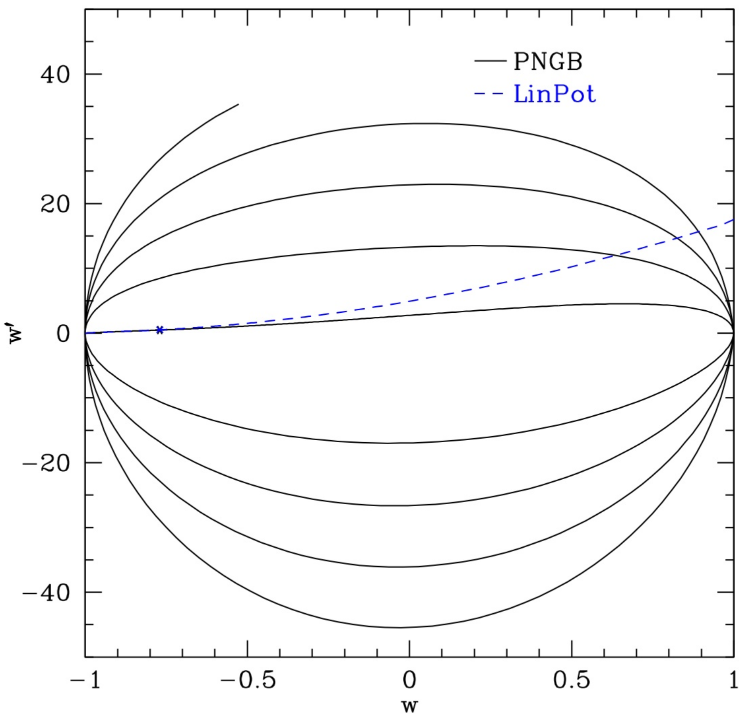


Asymptotic Behaviors

Asymptotic behaviors can be physically interesting.
Solve for critical points $x'(x_c,y_c)=0$, $y'(x_c,y_c)=0$.
Check stability by sign of eigenvalues $\delta p'=\mathcal{M}p$. $p=\{x,y\}$

Copeland, Liddle, Wands 1998
Phys. Rev. D 57, 4686

Relevant to fate of universe.



Crossing w=-1:

$$y = \frac{\kappa\sqrt{V}}{\sqrt{3}\,H} \quad \text{so} \quad y_c' = 0 \Rightarrow$$

$$\frac{V'}{V} = 2\frac{H'}{H} \equiv -3(1+w_{\rm tot})$$

Phantom fields roll up potential so $V'>0$, so $w_{tot}^{\infty}<-1$. Cannot cross w=-1 even with coupling. Quintessence can cross with coupling since $w<w_{tot}$.

Dark Energy Models

"Normal" potentials don't work:

$$V(\phi) \sim \phi^n$$

have minima (n even), and field just oscillates, leading to EOS

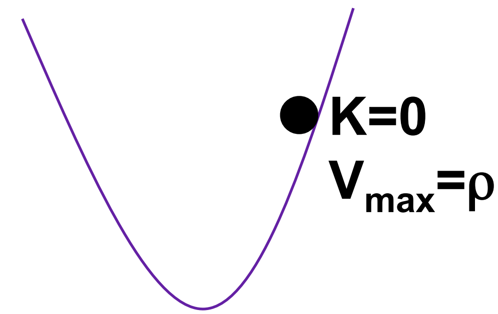
$$w = (n-2)/(n+2)$$

n	0	2	4	∞
w	-1	0	1/3	1

Oscillations

Oscillating field

$$w = (n-2)/(n+2)$$



Turner 1983

Take osc. time << H^{-1} and ρ constant over osc.

$$\langle \dot\phi^2 \rangle = \int dt\, \dot\phi^2 / \int dt = \int d\phi\, \dot\phi / \int d\phi / \dot\phi$$

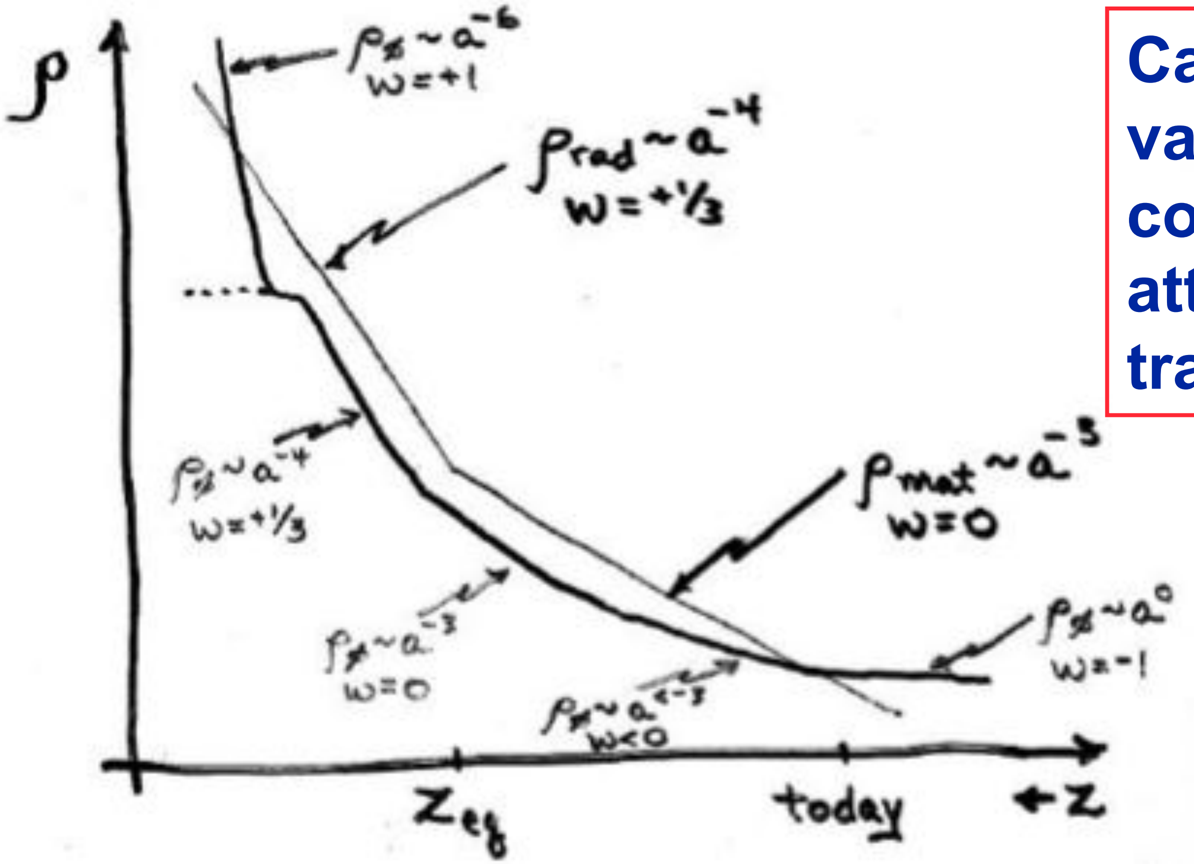
$$= 2\rho \int d\phi\, [1-V/V_{max}]^{1/2} / [1-V/V_{max}]^{-1/2}$$

If $V = V_{max}(\phi/\phi_{max})^n$ then

$$\langle w \rangle = -1 + 2 \int_0^1 dx\, (1-x^n)^{1/2} / \int_0^1 dx\, (1-x^n)^{-1/2}$$

$$= -1 + 2n/(n+2)$$

Tracking fields



Can start from wide variety of initial conditions, then join attractor trajectory of tracking behavior.

Criterion $\Gamma = VV''/(V')^2 > 1$, d *ln* $(\Gamma-1)/dt \ll H$.

However, generally only achieves $w_0 > -0.7$.

Successful model requires fast-slow roll.

Expansion History

Observations that map out expansion history a(t), or w(a), tell us about the fundamental physics of dark energy.

Alterations to Friedmann framework → *w(a)*

Suppose we admit our ignorance:

$$H^2 = (8\pi/3)\,\rho_m + \delta H^2(a)$$

gravitational extensions
or high energy physics

Effective equation of state:

$$w(a) = -1 - (1/3)\, d\mathit{ln}\,(\delta H^2) \,/\, d\mathit{ln}\, a$$

Modifications of the expansion history are equivalent to time variation w(a). Period.

Expansion History

For modifications δH^2, define an effective scalar field with

$$V = (3M_P^2/8\pi)\ \delta H^2 + (M_P^2 H_0^2/16\pi)\ [\ d\ \delta H^2/d\ \mathit{ln}\ a]$$

$$K = -\ (M_P^2 H_0^2/16\pi)\ [\ d\ \delta H^2/d\ \mathit{ln}\ a]$$

Example: $\delta H^2 = A(\rho_m)^n$

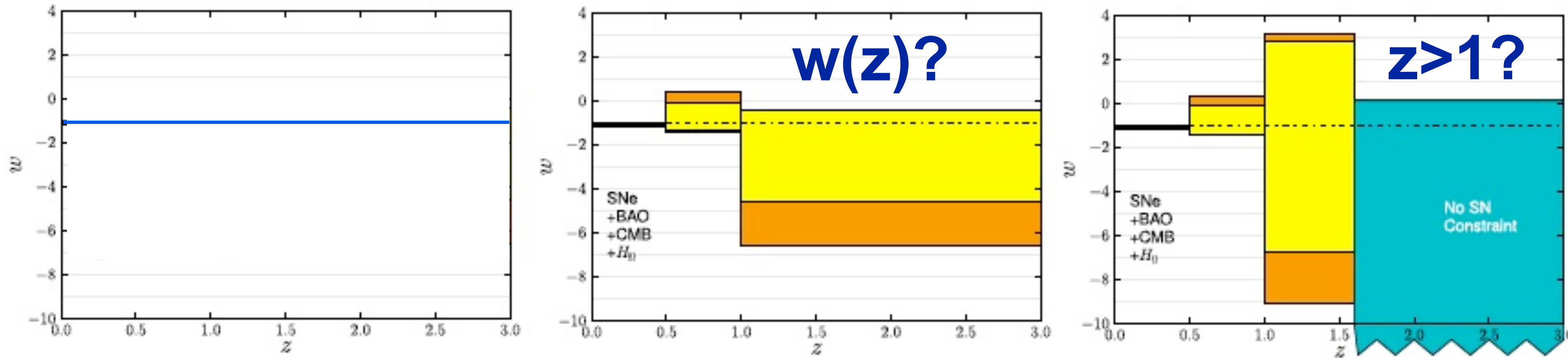
$$w = -1+n$$

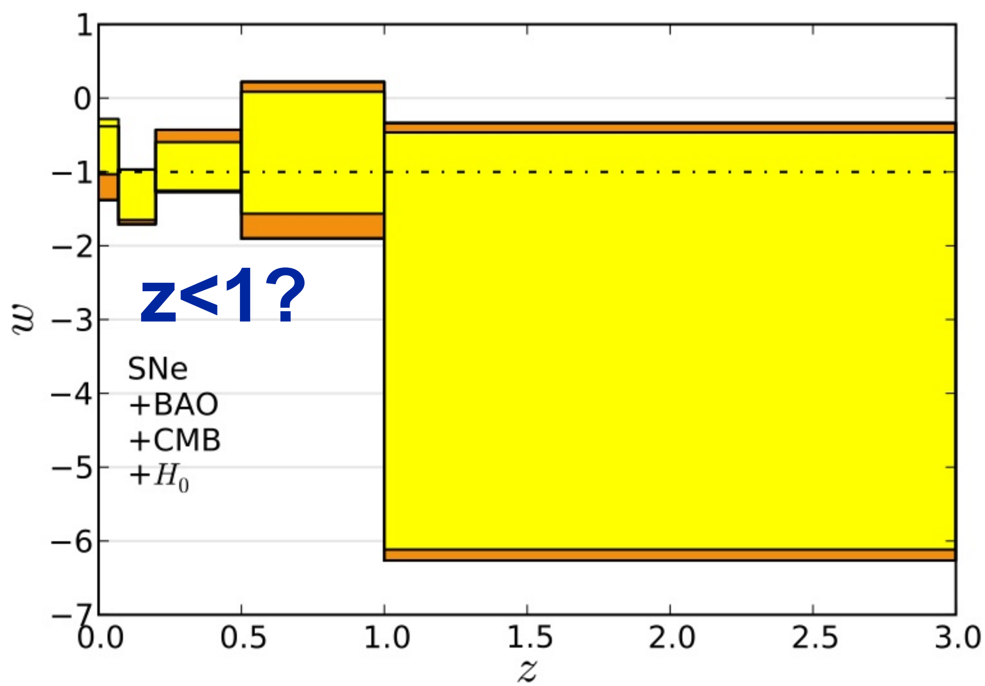
Example: $\delta H^2 = (8\pi/3)\ [g(\rho_m) - \rho_m]$

$$w = -1 + (g'-1)/[\ g/\rho_m - 1\ ]$$

Are We Done?

$$w = -1.013^{+0.068}_{-0.073} \quad \text{(stat+sys)}$$



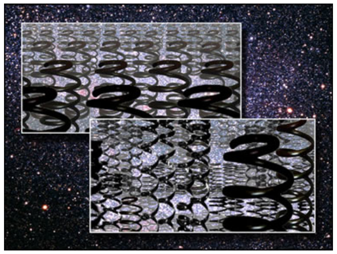


There is a long way to go still to say we have measured dark energy!

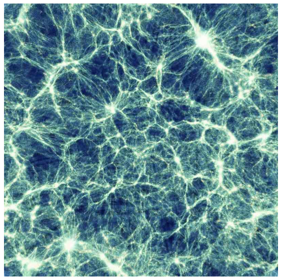
Dark Energy Properties

Dark energy is very much *not* the search for one number, "w".

Dynamics: Theories other than Λ give time variation $w(z)$. Form $w(z)=w_0+w_a z/(1+z)$ accurate to 0.1% in observable.



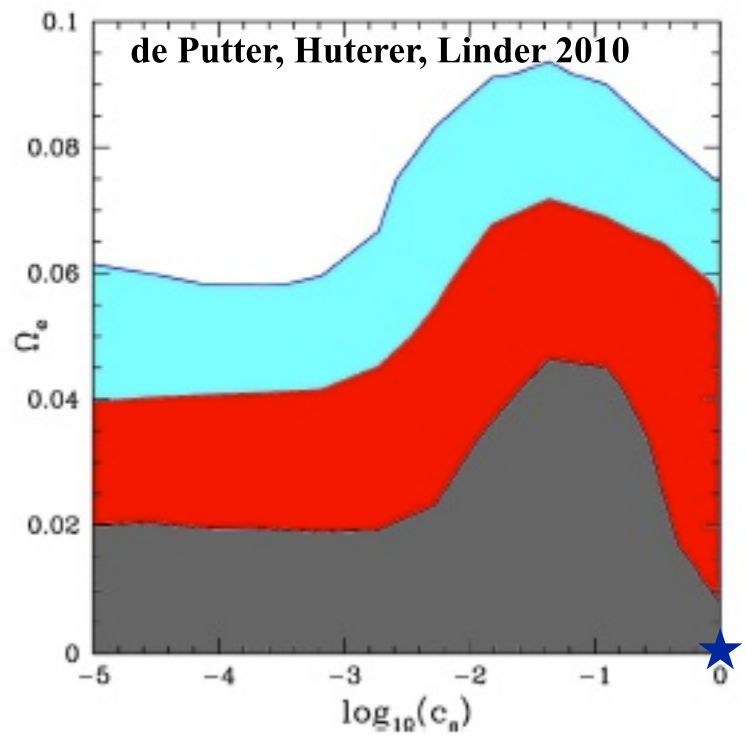
Degrees of freedom: Quintessence determines sound speed $c_s^2=1$. Barotropic DE has $c_s^2(w)$. But generally have $w(z)$, $c_s^2(z)$. Is DE cold ($c_s^2<<1$)? Cold DE enhances perturbations.



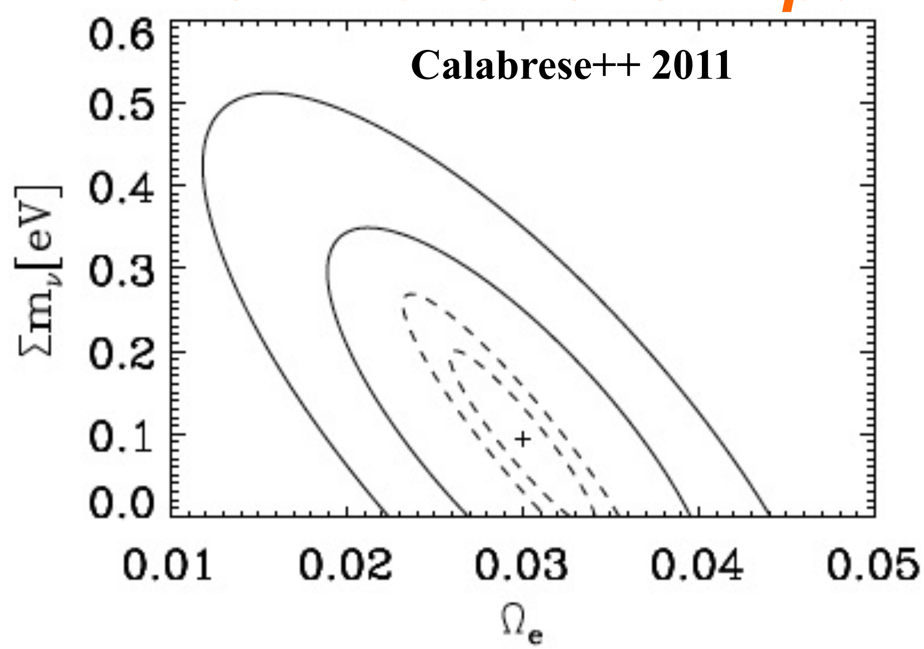
Persistence: Is there early DE (at $z>>1$)? $\Omega_\Lambda(z_{CMB})\sim10^{-9}$ but observations allow 10^{-2}.

The Speed of Dark

Current constraints on c_s using CMB (WMAP5), CMB × gal (2MASS,SDSS,NVSS), gal (SDSS).



Future constraints from Planck or CMBpol



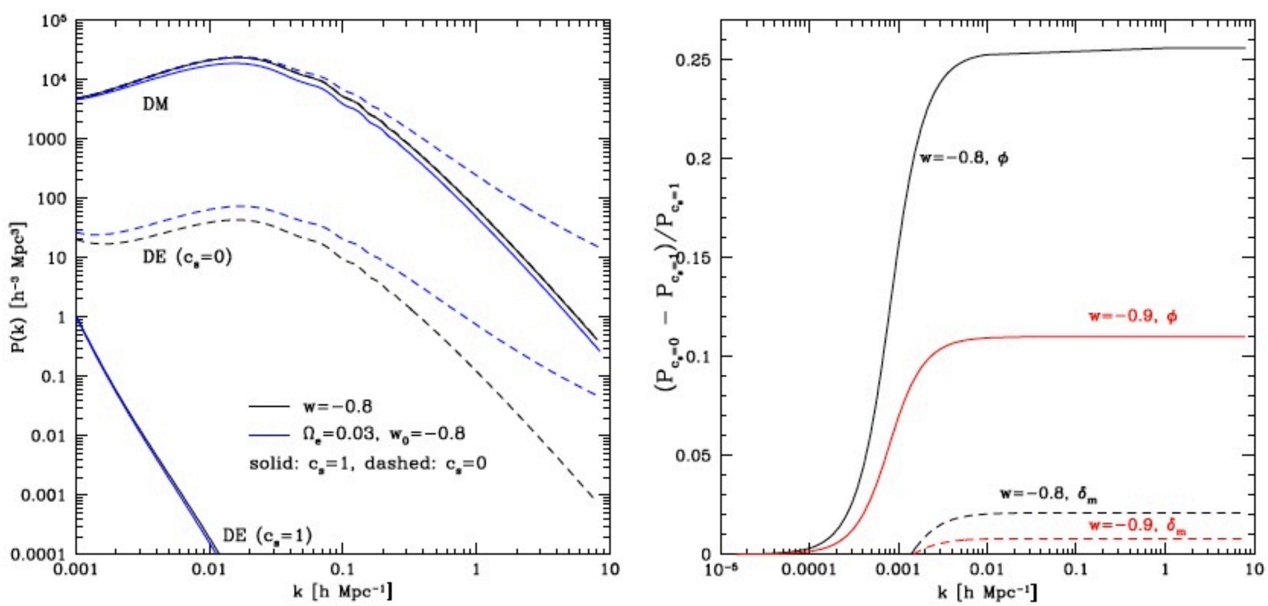
Best fit Ω_e=0.02, c_s=0.04, w_0=-0.95 but consistent with Λ within 68% cl.

"Early, Cold, or Stressed DE"
cf. generalized DE Hu 1998

Early, Cold, Stressed Dark Energy

Early DE **density** parametrized by **Doran & Robbers 2006** form. (Note $\Omega_\Lambda(z=10^3)\sim10^{-9}$.)

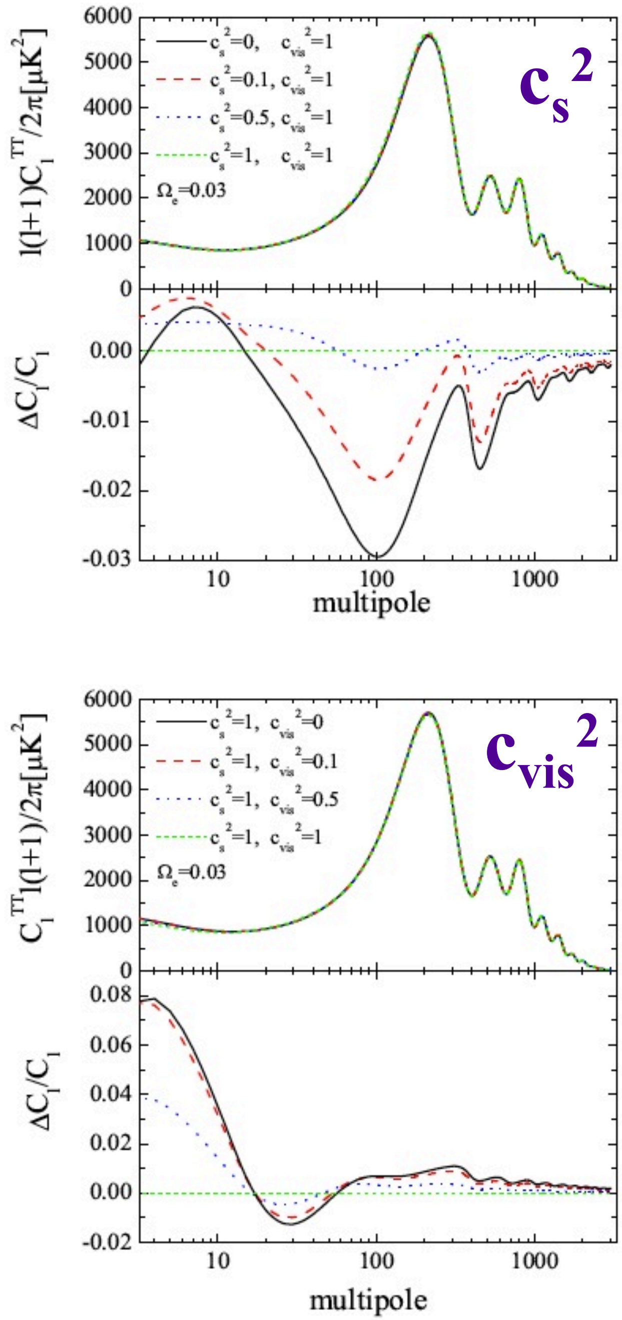
Perturbations by **sound speed** $c_s^2=dp/d\rho$. Quintessence has $c_s^2=1$. Largest effect for smallest c_s^2 – "cold dark energy".



Finally, **anisotropic stress** $c_{vis}\neq0$ (**Hu 1998**).

Early, Cold, Stressed Dark Energy

Perturbations enhanced by lowering **sound speed** c_s^2 (from 1) and suppressed by raising stress c_{vis}^2 (from 0).



Enhanced perturbations strengthen gravitational potential, so reduce photon Sachs-Wolfe power and enhance ISW.

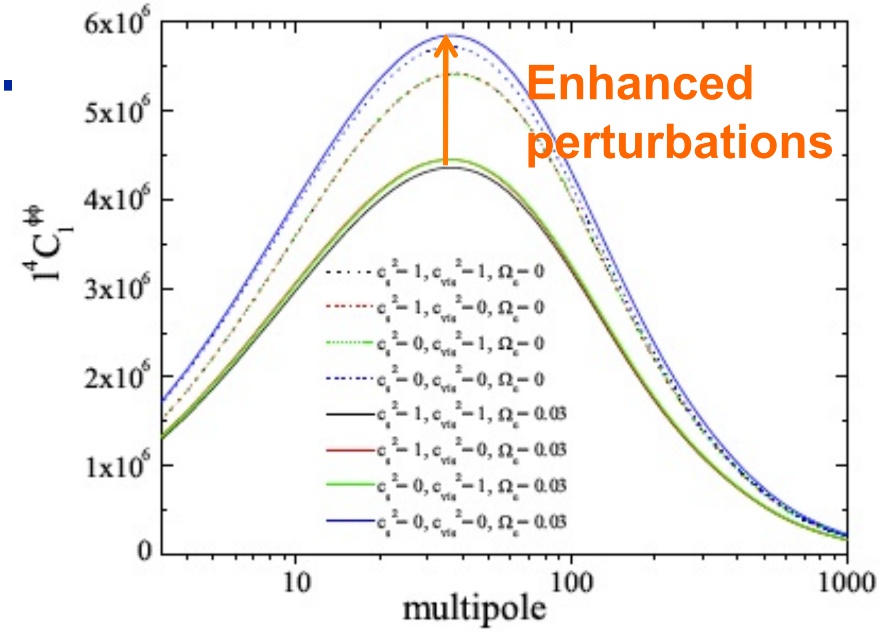
Calabrese, de Putter, Huterer, Linder, Melchiorri 2011

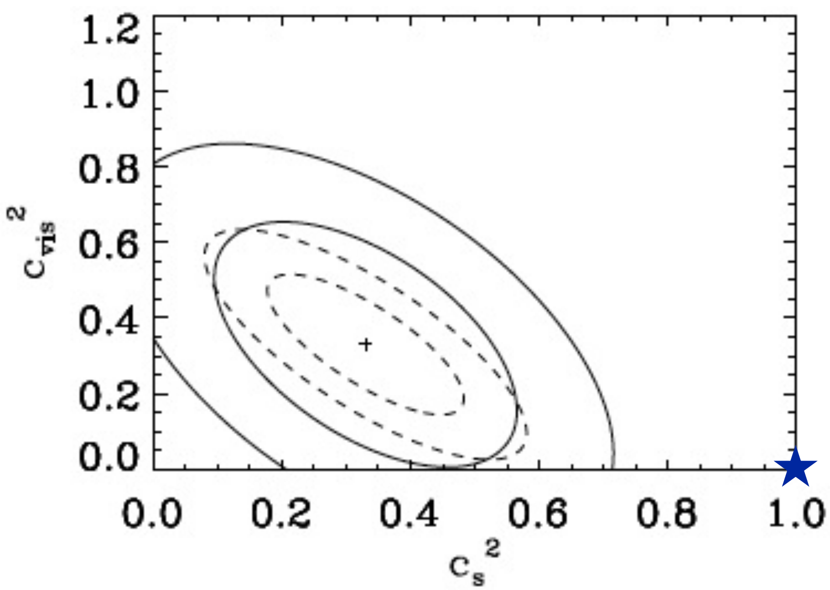
Early, Cold, Stressed Dark Energy

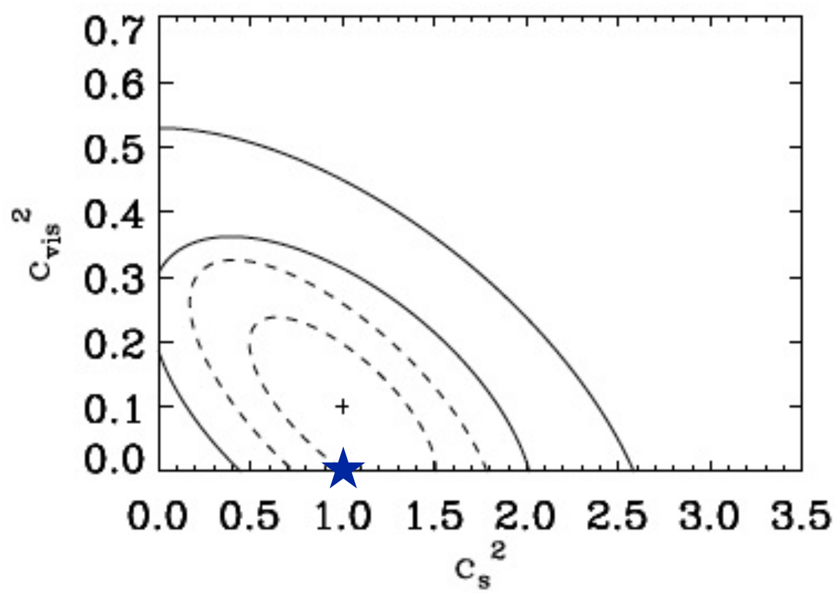
Also affects CMB lensing.

New degrees of freedom can be detected; testing consistency difficult.

Does not degrade other parameters.







Observational Leverage

Exercise 2.1: Solve the dynamics for a DBI scalar field

see Abramo & Finelli 2003

$$\mathcal{L}_\phi = -V(\phi)\sqrt{1-\dot{\phi}^2}$$

$$H^2 = \frac{\kappa^2}{3}\left[\rho_m + \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} - V(\phi)\sqrt{1-\dot{\phi}^2}\right]$$

For resources on dark energy as a field, see

Copeland, Sami, Tsujikawa 2006, *Dynamics of Dark Energy*
http://arxiv.org/abs/hep-th/0603057 and the references cited therein.