Energy-Momentum of the Vacuum

Quantum physics teaches us that the vacuum or ground state has energy and hence momentum. The principle of relativity teaches us that all observers must agree on the ground state, i.e. its energy-momentum tensor must be Lorentz invariant (e.g. having the same value for observers moving relative to each other). The only such Lorentz invariant is the Minkowski tensor $\eta^{\alpha\beta}$, so $T^{\alpha\beta} \sim \eta^{\alpha\beta}$.

a) Find the pressure p in terms of the energy density ρ.

b) Use the conservation equations ${T^{\alpha\beta}}_{,\beta} = 0$ for $\alpha = i$ to verify that the pressure is not only isotropic (as it must be for a perfect fluid) but homogenous.

c) Use the conservation equations ${T^{\alpha\beta}}_{,\beta} = 0$ for $\alpha = 0$ to find the evolution of the energy density with time.

The vacuum energy has to keep these properties even in general relativity, and is what Einstein called the **cosmological constant**.