

Power Spectrum of Perturbations

The power spectrum of density perturbations $\delta \equiv (\delta\rho/\rho)$, with index n , is

$$P_k \equiv |\delta_k|^2 \propto k^n.$$

a) Derive the expression for the rms density fluctuations on a comoving scale λ

$$\left(\frac{\delta\rho}{\rho}\right)_\lambda \sim k^{3/2} |\delta_k| \propto \lambda^p$$

and find the index p in terms of n .

b) Find the index m for the rms mass fluctuations on a comoving scale λ :

$$(\delta M/M)_\lambda \propto M^m.$$

What is the criterion on n such that structures grow in a top-down manner (large masses collapse first)? In a bottom-up manner? If the primordial spectrum has $n = 1$ and the CDM transfer function goes as $T(k) \sim k^{-2}$ due to the horizon entry effect, what m does this correspond to and what is the physical interpretation of the mass fluctuation spectrum?

c) The dynamics of the perturbations are driven by the fluctuations in the gravitational potential ϕ . Find the index f for the rms potential fluctuations

$$\delta\phi_\lambda \propto \lambda^f.$$

What is the physical significance of the scale invariant power spectrum index $n = 1$? Also explain the difference between a process being scale free and being scale invariant. Discuss which, if either, is Newtonian gravity.