CMB Anisotropies: Physical Processes

Radiation is not Jeans unstable so photon perturbations only arise from primordial matter perturbations – **passive fluctuations**.

Some models, e.g. topological defects or isocurvature models, continue to generate perturbations – **active fluctuations**.

Full treatment requires Boltzmann equation solver for photons, e.g. public domain CMBFAST. Simple treatment good fit for adiabatic perturbations, change only through $\delta T/T$.

Matter affects CMB through perturbations in density $\delta\rho/\rho$, velocity v, and gravitational potential ϕ. Velocity perturbations unimportant in standard model because they decay. Gravitational waves may contribute to ϕ.

Density Perturbations:

Adiabatic $\Rightarrow \eta = n_b/n_\gamma =$ constant

$$\frac{\delta n_\gamma}{n_\gamma} = \frac{\delta n_b}{n_b}$$

$$\Rightarrow \frac{\delta T}{T} = \frac{1}{3}\left(\frac{\delta\rho}{\rho}\right)_b$$

In terms of power spectrum

$$\left(\frac{\delta T}{T}\right)^2_{\delta\rho} = \frac{1}{9}\int d^3k\, P(k)$$

Velocity Perturbations:

Doppler shift gives dipole moment

$$\frac{\delta T}{T} = \frac{\vec{v}\cdot\hat{n}}{c}$$

In terms of power spectrum

$$\left(\frac{\delta T}{T}\right)^2_{v} = (Haf)^2\int d^3k\, P(k)\, k^{-2}\mu^2$$

where $f \equiv d\ln D/d\ln a \approx \Omega^{0.6}$ is linear growth rate.

Potential Perturbations:

Equivalence Principle teaches that photon gravitationally redshifts in regions of differing potential. But also affects time of "emission", i.e. last scattering. So

$$\frac{\delta T}{T} = \delta\phi + \frac{1}{T}\frac{dT}{dt}\delta t.$$

Now

$$\frac{dT}{T} = -\frac{da}{a} = -\frac{2}{3}\frac{dt}{t} = -\frac{2}{3}\delta\phi$$

where the last equality is the photon frequency's gravitational redshift. So

$$\frac{\delta T}{T} = \frac{1}{3}\delta\phi.$$

In terms of power spectrum

$$\left(\frac{\delta T}{T}\right)^2_\phi = \frac{9}{4}H^4 a^2 \int d^3k\, P(k)\, k^{-4}$$

using Poisson's equation.

This is sometimes called the *Sachs-Wolfe effect*, but that generally involves the full integral of the potential along the photon path. In the Einstein-deSitter case only the endpoint effect given above enters.

We can estimate the relative importance of these three effects. The temperature perturbations in Fourier space are

$$\mathcal{T}^2(k) = \left[(\delta_\rho + \delta_\phi)^2(k) + \delta_v^2(k)\,\mu^2\right]\ \Delta_k^2(z_{ls})$$

where $\mu = \hat{k}\cdot\hat{n}$ and

$$\delta_\rho = \frac{1}{3}$$

$$\delta_v \sim \frac{1}{kL}$$

$$\delta_\phi \sim -\frac{1}{(kL)^2}$$

It is clear that on

- Large scales (small k): Sachs-Wolfe effect dominates
- Medium $\approx$cluster scales: Doppler effect of velocity flows is important
- Small $\approx$galaxy scales (large k): density perturbations dominate

In next lecture will translate these to angular scales and multipoles.

Sachs-Wolfe Effect

Equivalence Principle gives $\delta T/T$ from $\delta\phi$. General relativity calculation of photon geodesic equation from emitter to observer gives

$$\frac{\delta T}{T} = -\int_e^o dt\,\frac{\partial\phi(\vec{x},t)}{\partial t}.$$

In linear perturbation theory, Einstein-deSitter universe has $\delta_k \propto a$ so Poisson's equation

$$\phi_k \propto -4\pi k^{-2}\rho a^2\delta_k \propto a^0.$$

Therefore only surface terms of integral survive – **endpoint Sachs-Wolfe** effect.

Other Sachs-Wolfe contributions are hallmarks of breakdown of linearity or of Einstein-deSitter.

• **Integrated Sachs-Wolfe Effect:** (ISW effect) At early times, near last scattering, radiation energy density still affects expansion somewhat, so $\delta \propto a$ not exact. At late times, near the present, an open universe also causes deviations from $\delta \propto a$ (called **late time SW effect**).

• **Rees-Sciama Effect:** Nonlinear structure, e.g. clusters, break $\delta \propto a$. Photons passing through static potential well suffer no redshift, but expansion stretches potential during passage.

$$\frac{\delta T}{T} \approx \frac{d\phi}{dt}\Delta t \sim \phi \frac{L}{c\tau}$$

where L is size of cluster, τ is cluster evolution timescale.

Since $L < c\tau$ then $(\delta T/T)_{RS} < (\delta T/T)_{SW}$.

Call perturbations at last scattering **primary** anisotropies, more recent ones **secondary**.

To estimate magnitude relate velocity dispersion to potential by $\phi \sim (v/c)^2$.

$$\left(\frac{\delta T}{T}\right)_{SW} \approx \frac{1}{3}\left(\frac{v}{c}\right)^2 \approx 3 \times 10^{-6} \left(\frac{v}{1000\,\mathrm{km\ s^{-1}}}\right)^2 .$$

For Rees-Sciama $L/(c\tau) \sim v/c$ then

$$\left(\frac{\delta T}{T}\right)_{RS} \approx (v/c)^3 \approx 10^{-7.5} \left(\frac{v}{1000\,\mathrm{km\ s^{-1}}}\right)^3 .$$

The largest voids are about 60 h^{-1}Mpc so $v/c = HR = 1/50$ and

$$\left(\frac{\delta T}{T}\right)_{RS} (\text{void}) < 10^{-5}.$$

So we want to measure temperature perturbations of order 10^{-5} or smaller. This isn't easy, but the detailed structure of the anisotropies on different angular scales reveals the origin of structure and the process of structure formation.