Standard Candle Test

The best and most current measurements of the energy density of the universe, in the form of the deceleration parameter q_0, come from using high redshift supernovae as standard candles. For standard candles the observed magnitude is related to the luminosity distance by $m(z) \sim 5\log[H_0 r_l(z)]$, where

$$H_0 r_l(z) = (1+z)(1-\Omega)^{-1/2} \sinh\left[(1-\Omega)^{1/2} \int_1^{1+z} dy\, [\Omega_m y^3 + \Lambda - (\Omega - 1)y^2]^{-1/2}\right],$$

for universes with matter plus cosmological constant, $\Omega = \Omega_m + \Lambda$.

a) Show that in a low redshift expansion neither Ω nor Λ enters at $\mathcal{O}(z)$ and they only enter in the combination q_0 at $\mathcal{O}(z^2)$. Thus the curvature of the magnitude-redshift plot (called the Hubble diagram) probes the deceleration of the universe.

b) Plot the deviation $m(z) - m_f(z)$ from the magnitude relation of a fiducial model $(\Omega_m, \Lambda) = (0.2, 0)$, for $(\Omega_m, \Lambda) = (1, 0)$, (0.25, 0.75), (0.4,0.6), (0.4, 0) out to $z = 2$. Which of these models comes closest to the observed data points of $m - m_f = 0.1 \pm 0.15$, 0.2 ± 0.15, 0.4 ± 0.35 at $z = 0.1$, 0.6, 1?