Time-varying G and Type Ia Supernovae

Eric V. Linder

Since the premier method of probing the nature of the dark energy accelerating the cosmic expansion is Type Ia supernovae (SN), it is important to make sure that the SN themselves are not affected by dark energy. In particular, if the cosmic acceleration arises from a modification of gravity, this might alter the structure and hence brightness of SN. Here I present a brief calculation, extending and correcting the work of Gaztañaga et al. 2002, showing that such gravitational effects on SN are not of concern.

I keep the computation very simple to show essential points. The gravitational influence is treated as a time varying Newton's constant G, as is generic for scalar-tensor theories. The luminosity of a SN is powered by the Nickel-56 mass created, which is roughly proportional to the Chandrasekhar mass. This in turn depends on the gravitational strength as $G^{-3/2}$, so a SN at redshift z has luminosity

$$L(z) \sim M_{\rm Ch} \sim G^{-3/2}.$$

SN measurements are interpreted in terms of the Hubble diagram of the magnitude-redshift relation through the corrected peak magnitude m. The one parameter correction involves the "stretch" factor of the time dependence of the flux (called the light curve). This time scale stretch $s-1$, arising from opacity effects in the stellar atmosphere, and hence the energy released to excite atomic transitions, is proportional to $M_{\rm Ch}^{1/2} \sim G^{-3/4}$. Finally, the luminosity distance is the integral over the inverse Hubble parameter, which is proportional to $G^{-1/2}$.

Putting this all together,

$$m \sim -2.5 \log L + \alpha(s-1) + 5 \log d_l,$$

where $\alpha \approx 1.5$ is the coefficient of the stretch correction. We can now evaluate how a change ΔG from redshift z to today will affect the SN magnitude on a Hubble diagram. Since d_l is an integral quantity, we approximate the effect of varying G by a simple average at z and today ($z=0$), i.e. $\Delta G/2$.

The result is that the magnitude on the Hubble diagram is affected as

$$\Delta m \sim -\frac{1}{8}\frac{\Delta G}{G}.$$

The SNAP mission aims to measure SN out to $z=1.7$ to below a systematic (coherent bias) level of 0.02 magnitudes. Thus for a consistent analysis we require that $|\Delta G/G| < 0.16$ out to $z=1.7$.

Violation of this bound requires a relatively huge variation. Current constraints on $\Delta G/G$ are on the order of 2×10^{-5}, if one uses solar system constraints on the Jordan-Brans-Dicke parameter, or < 0.003 if one uses primordial nucleosynthesis limits (see Appendix for details). The latter limit is equivalent to $|\dot{G}/G| < 2\times 10^{-13}\,{\rm y}^{-1}$.

Thus, variation of Newton's constant, whether induced by dark energy or otherwise, does not affect SN observations at the sensitivity level of anticipated precision measurements.

Appendix. Limits on $\dot{G}$

Here I give a brief overview of the observational constraints on time varying G.

Scalar-tensor gravity

Within scalar-tensor theory one generically has

$$\dot{G}/G \approx -\dot{F}/F,$$

where $F(\phi)$ is the coupling function of the scalar field ϕ to the Ricci curvature R. That is, the factor $R/(8\pi G)$ in the Einstein-Hilbert action is replaced by $F(\phi)\,R$. We can write $\dot{F} = F_{,\phi}\dot{\phi}$ and using the slow roll approximation find that

$$\dot{\phi} = F_{,\phi}R/(6H) = HF_{,\phi}(1-q),$$

where q is the deceleration parameter. Thus,

$$\dot{G}/G = -(F_{,\phi}^2/F)H(1-q) \equiv -H(1-q)/\omega_{JBD},$$

where ω_{JBD} is the Jordan-Brans-Dicke parameter.

Since the average value of $1-q$ over the period between today and $z = 1.7$, when the SN measurements are probing the dark energy, is near unity, and $H = d\ln a/dt$, then we have

$$\Delta G/G \approx (1/\omega_{JBD})(-\Delta \ln a).$$

For $z = 0 - 1.7$, $\Delta \ln a = -1$, so the fractional change in G is just the inverse Jordan-Brans-Dicke parameter. This has been stringently constrained by solar system tests (see Bertotti, Iess, and Tortora 2003), so

$$\Delta G/G < 2.5 \times 10^{-4}.$$

Primordial nucleosynthesis

For primordial nucleosynthesis limits, we realize that a change in G gives a change in the expansion rate during nucleosynthesis, changing the element abundances. A quick treatment of this is in Chapter 6.6.4 of First Principles of Cosmology (Linder 1997), leading to a limit $\Delta G/G < 0.006$. For more on non-standard expansion during nucleosynthesis, see Kaplinghat & Carroll 2002, and for a rigorous treatment of varying G constraints from nucleosynthesis see Clifton, Barrow, and Scherrer 2005. Their limit corresponds to $\Delta G/G < 0.003$.